Callands Community Primary School

Calculation Policy

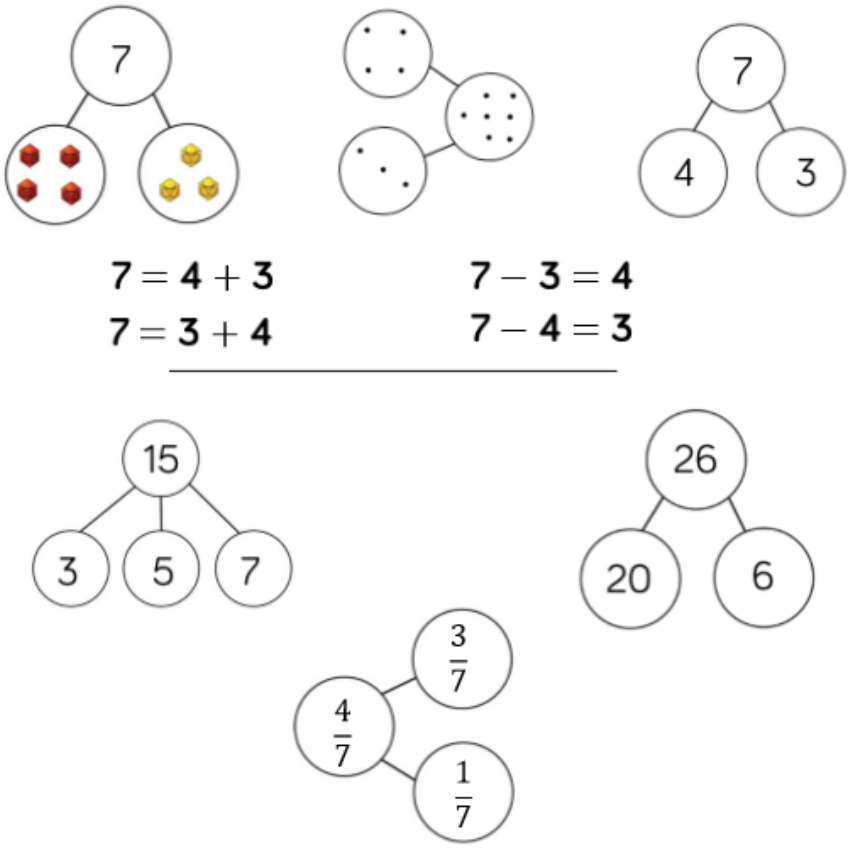
May 2021

Addition & Subtraction Strategies

Modelling

The following methods of modelling are used when teaching addition and subtraction. There is an example of each technique taught and an explanation as to the benefits of each method.

Part-Whole Model



Benefits of this model:

This supports children's understanding of aggregation and partitioning.

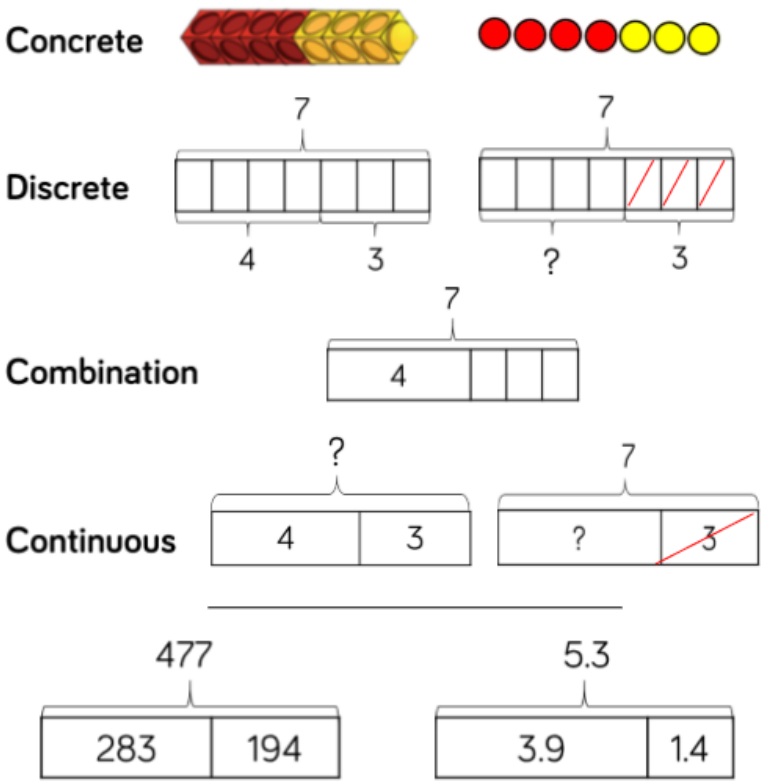
When the parts are complete and the whole is empty, aggregation is used to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, then partitioning (which is a form of subtraction) is used to find the missing part.

These models can be used to partition into 2 or more parts or to help visualise partitioning an amount into place value amounts instead of only using columns to represent this.

In KS2, these models are also used to add and subtract fractions, decimals and percentages.

Bar Model (Single)



Benefits of this model:

This model is a different form of part-whole model that represents calculations.

Cubes and counters can be used in a line as a concrete representation of this type of bar model. This is particularly useful for EYFS and KS1.

In EYFS and KS1, the discrete bar model is a good starting point with smaller numbers where each box represents one whole.

From KS1 to LKS2, the combination bar model supports children to calculate by counting on from the larger number and is a good stepping stone towards the continuous bar model.

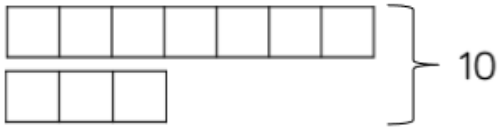
In LKS2 to UKS2, the continuous bar model is useful for a range of values; where each rectangle represents a number.

In UKS2, these models are used to represent larger numbers, decimals and fractions.

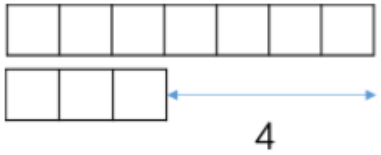
Question marks are used to indicate the value to be found.

Bar Model (Multiple)

Discrete

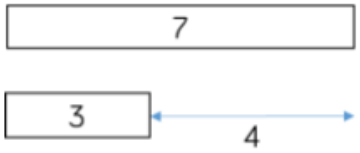


$7 + 3 = 10$

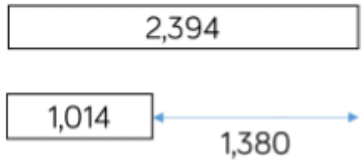


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits of this model:

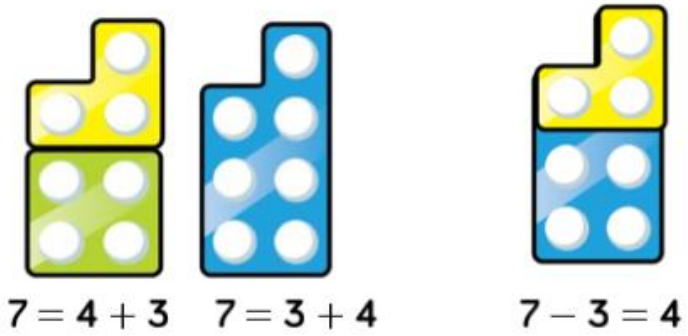
This method is a good way to compare quantities whilst still representing calculations.

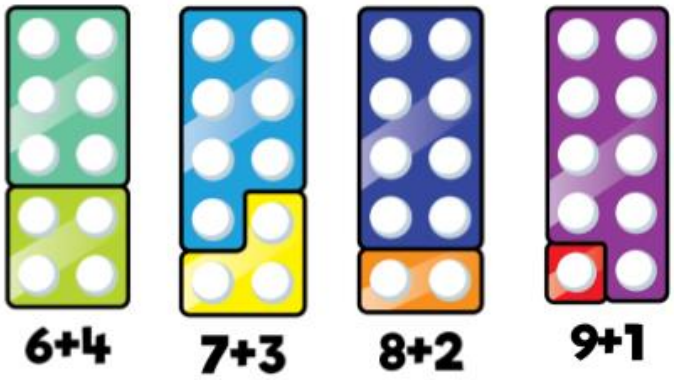
Two or more bars are drawn with a bracket labelling the whole; usually to the right-hand side of the bars. Smaller numbers are represented with a discrete bar model and larger numbers use continuous bar models.

These models can also be used to represent the difference in subtraction by using an arrow.

When using smaller numbers, cubes and discrete bars to find the difference which supports counting on.

Number Shapes





Benefits of this model:

This method is a good way to explore aggregation, partitioning and number bonds.

When adding numbers, it is possible to see how the parts come together to make a whole.

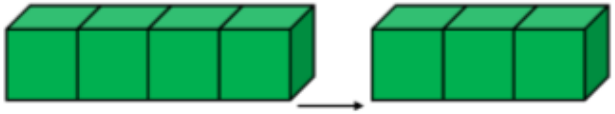
When subtracting numbers, children can start with the whole then place the part on top to see the missing amount.

This method allows children to work systematically to find number bonds.

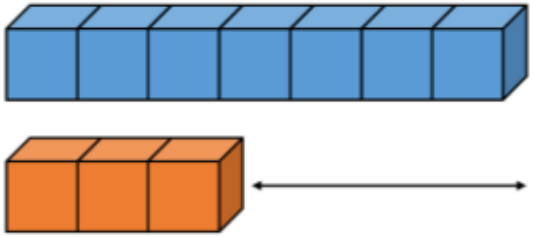
Cubes

$7 = 4 + 3$

$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

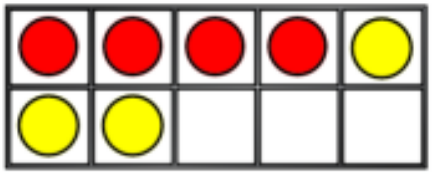
Benefits of this method:

This can be useful to support with addition and subtraction of one-digit numbers but are less efficient when working with larger numbers.

When adding numbers, it is possible to see how the parts come together to make a whole. Two different colours can also be used to represent the individual numbers before putting them together to create the new whole.

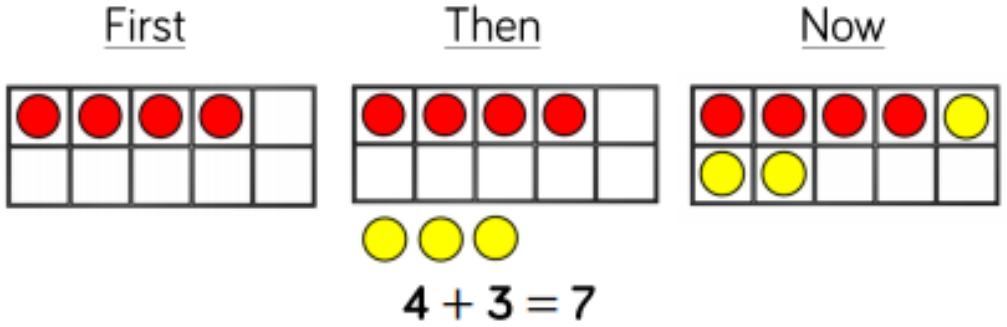
When subtracting numbers, children are able to start with the whole and then remove the number of cubes they are subtracting in order to see the answer (referred to as reduction or take away).

Ten Frames (within 10)

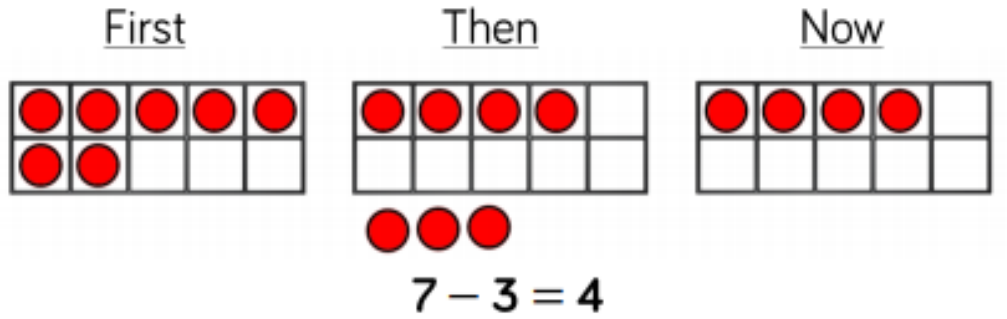


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.



$4 + 3 = 7$



$7 - 3 = 4$

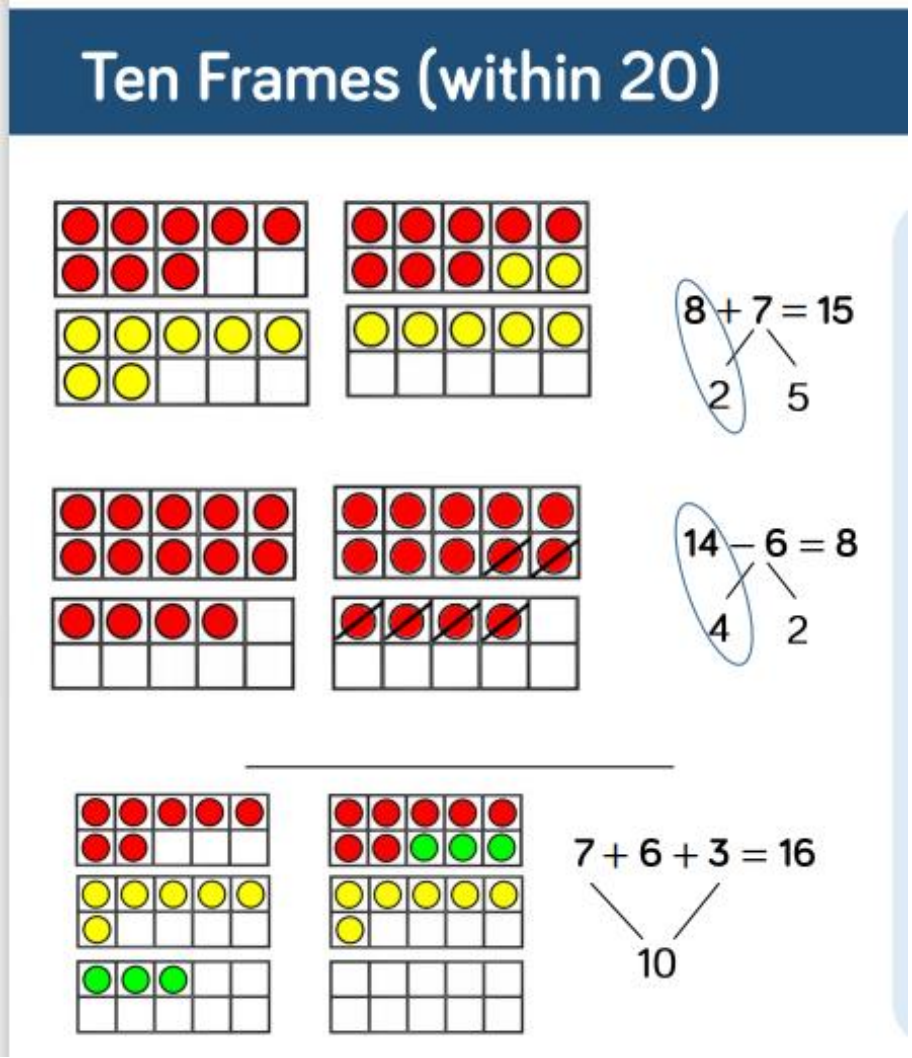
Benefits of this method:

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Tens frames and number stories are a useful method for this.



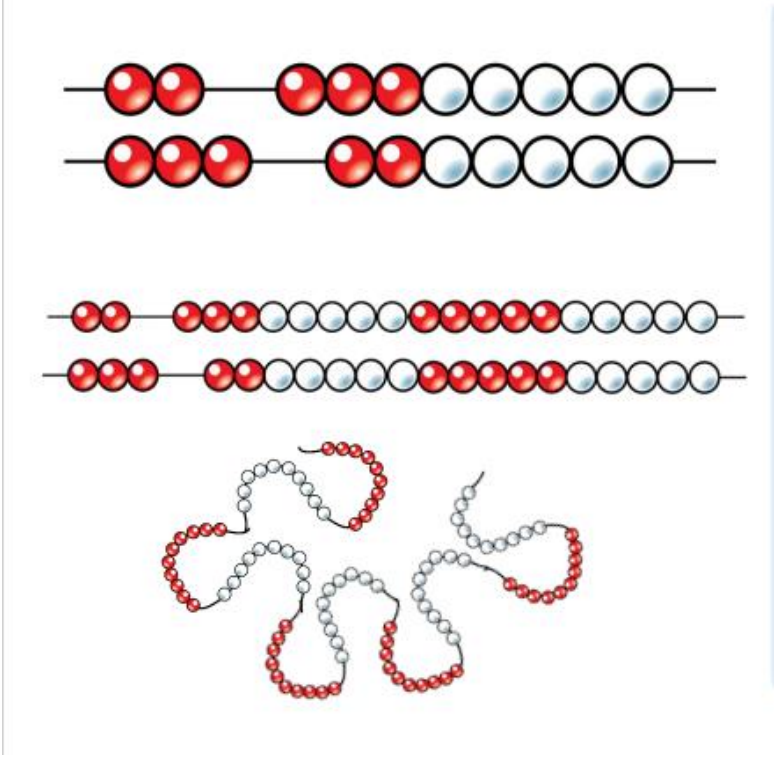
Benefits of this method:

When adding two single digits, children can make each number on a separate tens frame before moving one number to make 10 on one of the tens frames. This supports children to see how they partitioned one of the numbers to make 10, and makes links to effective mental methods.

When subtracting a one-digit numbers form a two-digit number, firstly make the larger number on 2 tens frames and then remove the smaller number.

When adding three single-digit numbers, children can make each number on a separate tens frame before considering which order to add the numbers in.

Bead Strings



Benefits of this method:

Different sized bead strings can support children at different stages of addition and subtraction.

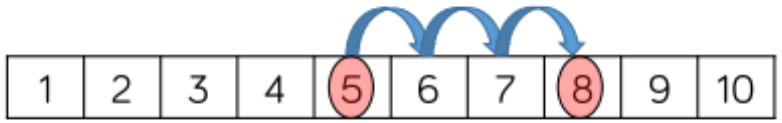
Bead strings strings to 10 are obviously good for practising number bonds to 10 as the beads can be moved one at a time.

Bead strings to 20 are similar but they also group beads in 5's. This also helps link number bonds to 20 back with number bonds to 10.

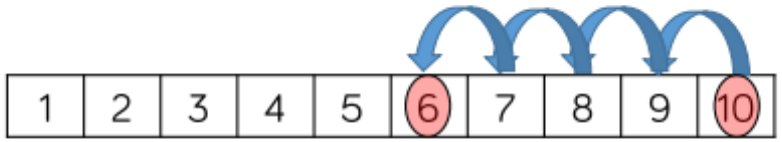
Bead strings to 100 help with bonds to 100 and also count in patterns of 10. This also demonstrates adding the next ten on a number line.

Number Tracks

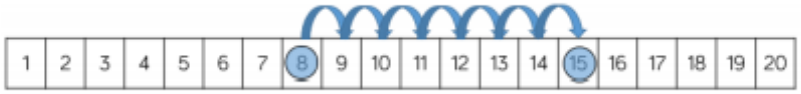
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$



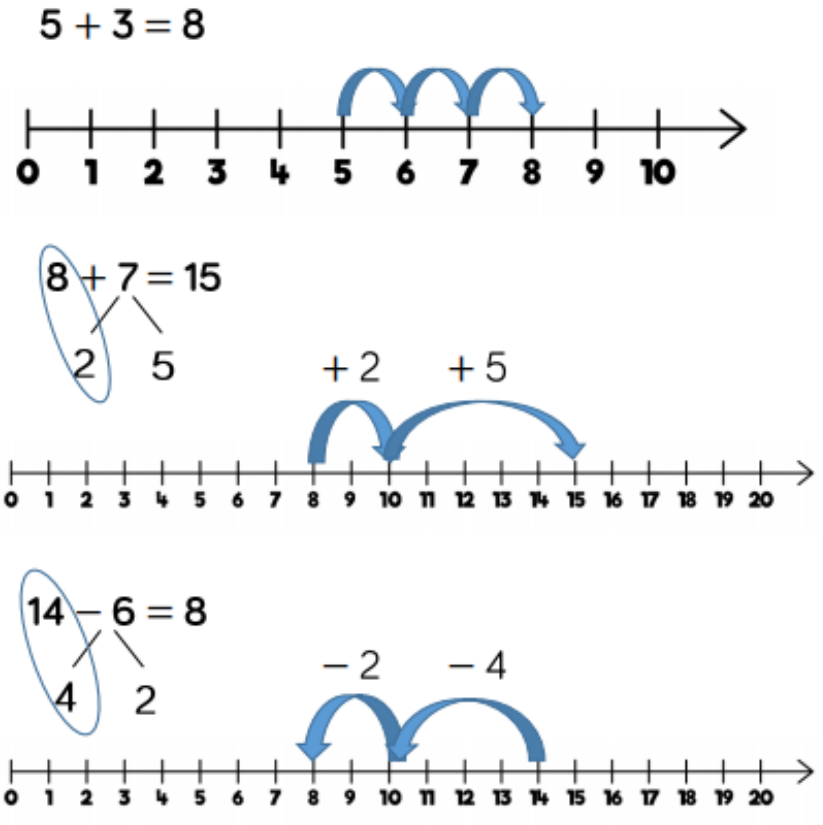
Benefits of this method:

Number tracks are useful to support children in their understanding of augmentation and reduction.

This method helps with counting on for addition and subtraction/difference.

Playing board games will help children become confident with number tracks.

Number Lines (labelled)



Benefits of this method:

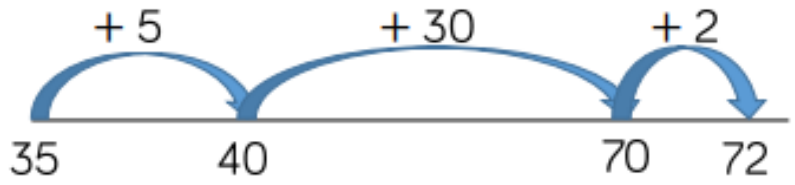
Labelled number lines support children in their understanding of augmentation and reduction.

This skills links directly with the number track.

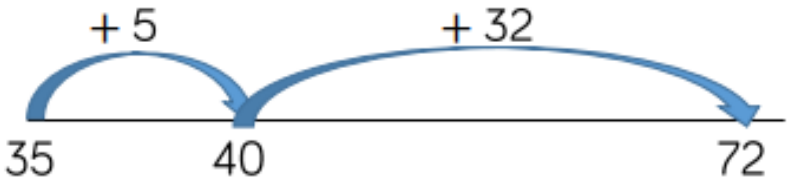
Progressing further, this supports more complex addition and subtraction as children can jumping to the nearest ten to the number. This partitioning method can also be shown using a tens frame.

Number Lines (blank)

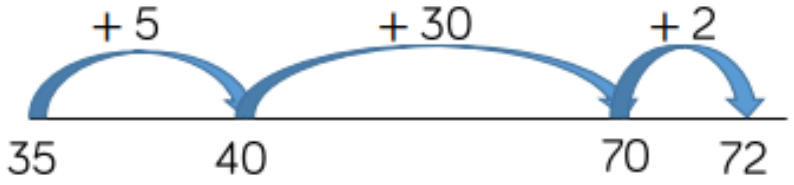
35 + 37 = 72



35 + 37 = 72



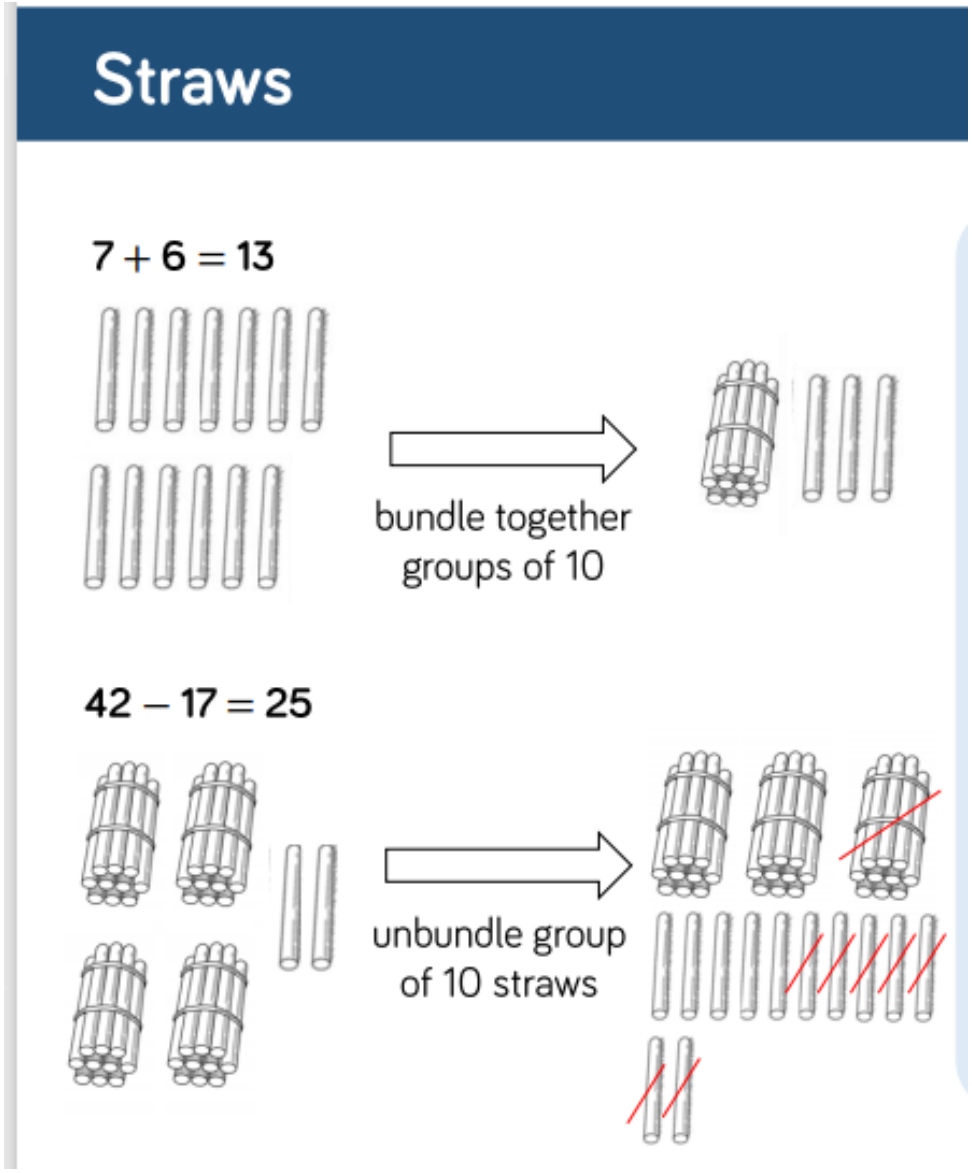
72 − 35 = 37



Benefits of this method:

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

This method enables children to make larger jumps and calculate more efficiently.



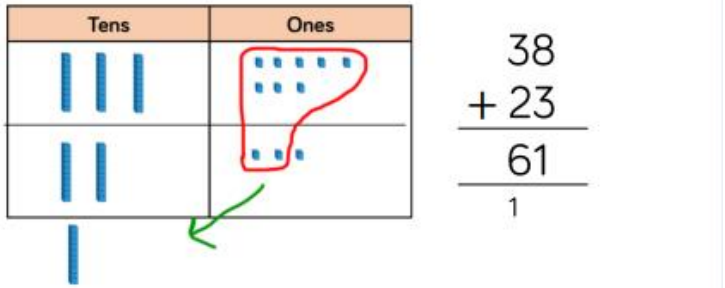
Benefits of this method:

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

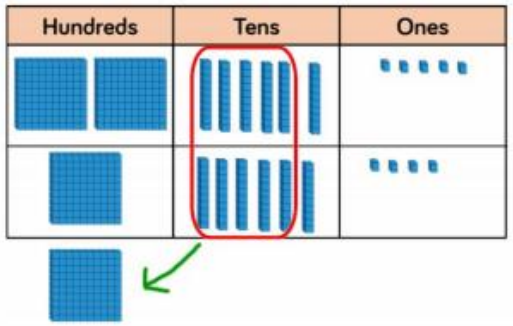
When adding, children can bundle a group of 10 straws to represent exchange from ten ones to 1 ten.

When subtracting, children can unbundle a group of 10 straws to represent exchange from1 ten to ten ones.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Benefits of this method:

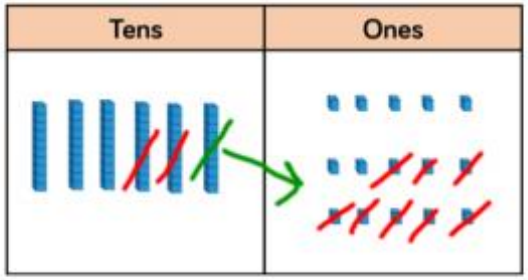
Using base ten or Dienes is an effective way to support children's understanding of column addition. The written method must be written alongside, so that clear links are made.

Children should first add without exchange and then move on to exchange.

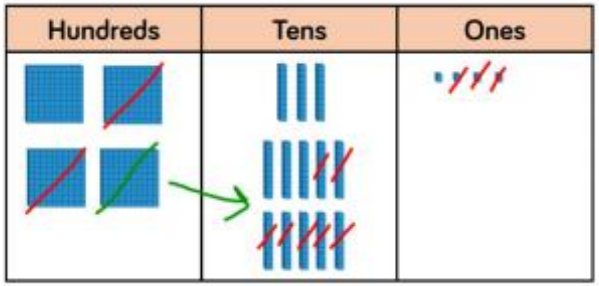
When adding always start with the ones column.

This model is efficient with up to 4 digit numbers. Place value counters are more efficient with larger numbers and decimals.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 28 \\ \hline 37 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}5 \\ -\ 273 \\ \hline 262 \end{array}$$

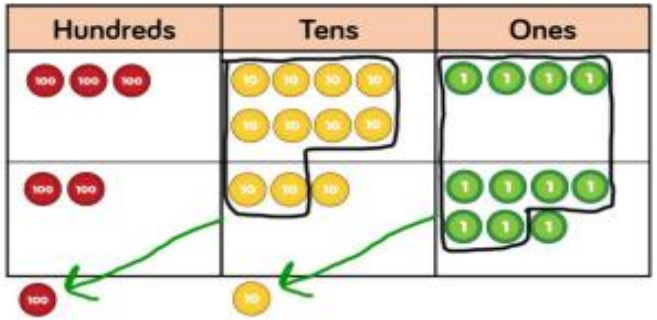
Benefits of this method:

Using base ten or Dienes is an effective way to support children's understanding of column subtraction. The written method must be written alongside, so that clear links are made.

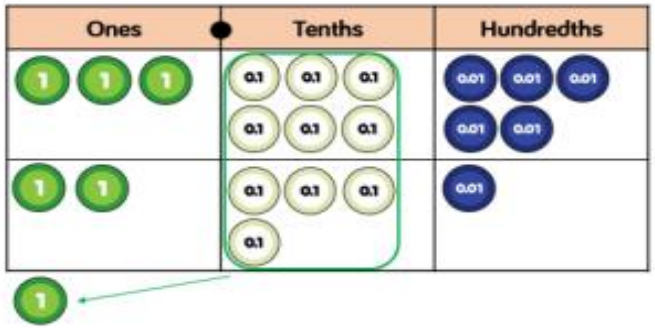
Children should first subtract without exchange and then move on to exchange.

This model is efficient with up to 4 digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ \ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

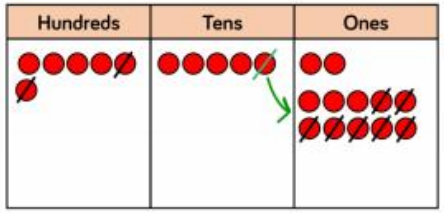
Benefits of this method:

Using place value counters is an effective way of supporting column addition. It is important that the children write the calculation alongside the grid so clear links are made.

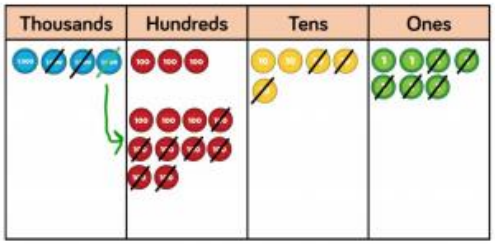
Children should first add without exchange before moving onto exchange.

Coins can also be used when working with money.

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\not{6}\,{}^{1}\not{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Benefits of this method:

Using place value counters is an effective way of supporting column subtraction. It is important that the children write the calculation alongside the grid so clear links are made.

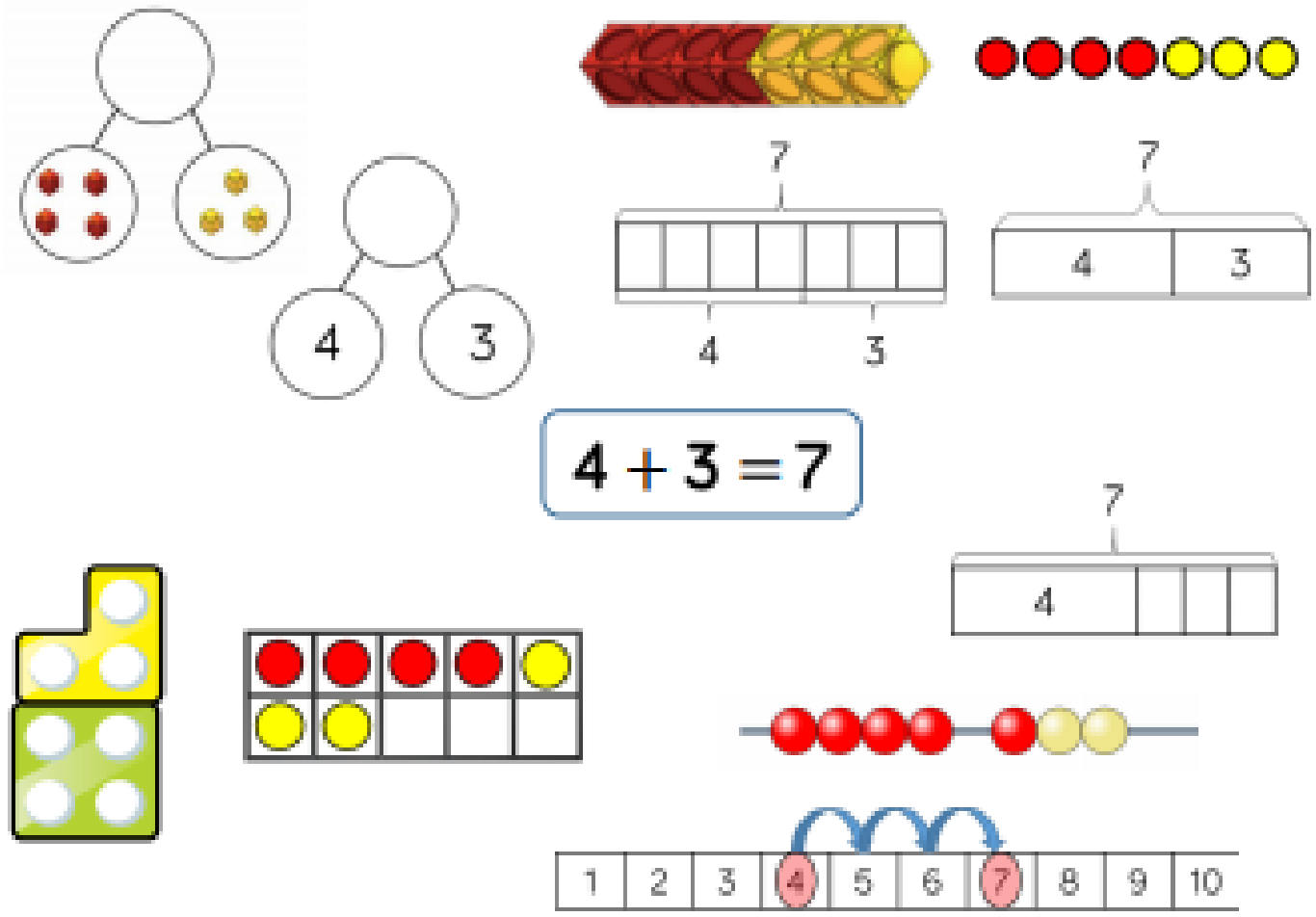
Children should first subtract without exchange before moving onto exchange.

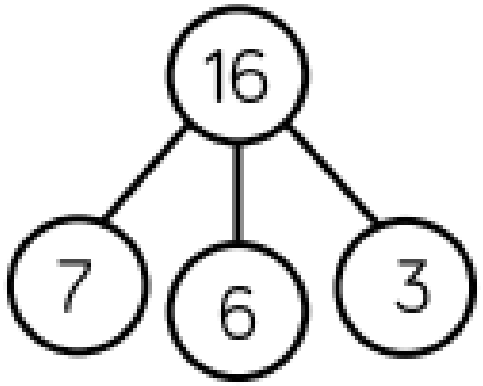
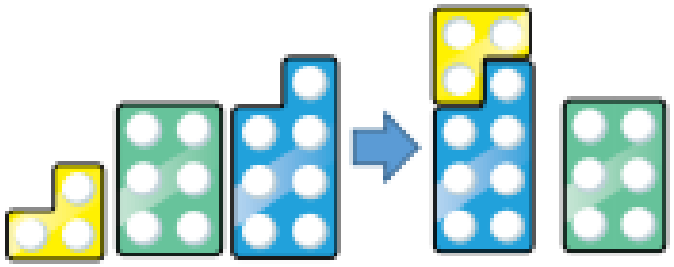
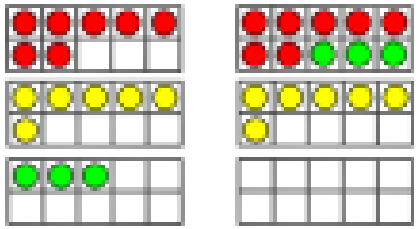
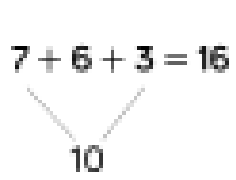
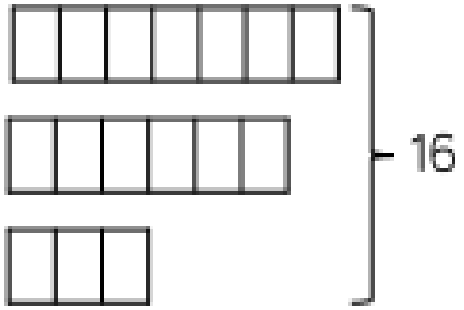
Coins can also be used when working with money.

ADDITION

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

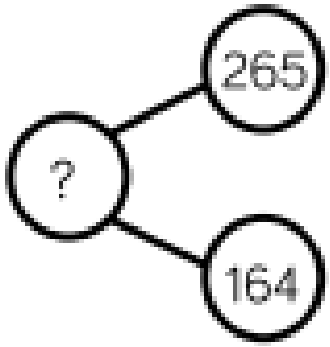
Addition

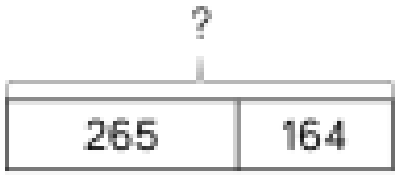
Skill: Add 1-digit numbers within 10	Year: 1
	When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add three 1-digit numbers	Year: 2
  $7 + 6 + 3 = 16$   	When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

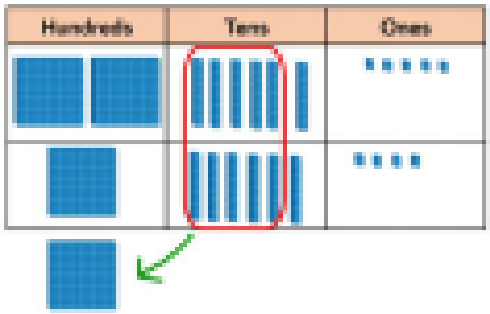
Skill: Add numbers with up to 3 digits

Year: 3

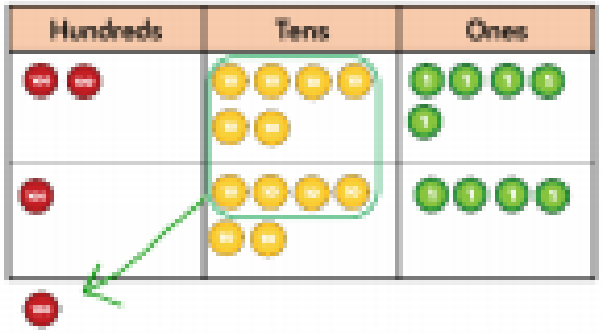




$$265 + 164 = 429$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ {\scriptstyle 1\ \ \ \ } \end{array}$$



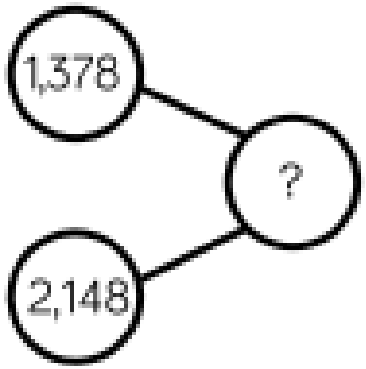
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

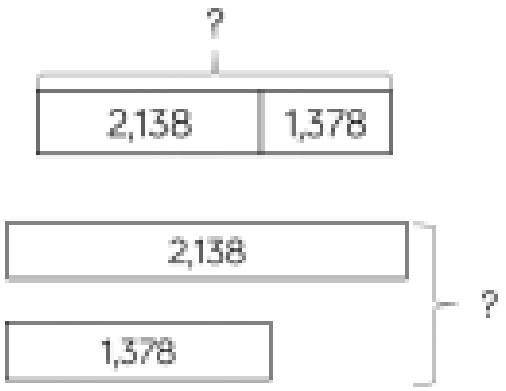
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

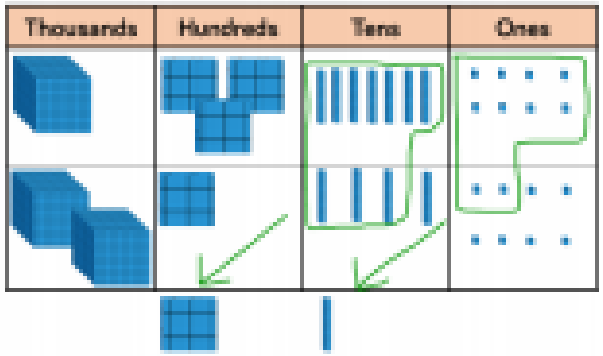
Year: 4

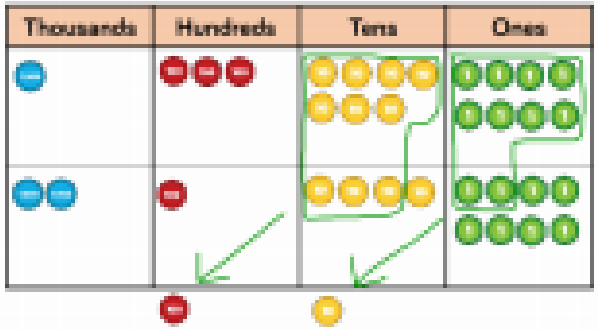




	1	3	7	8
+	2	1	4	8
	3	5	2	6
	1	1		

1,378 + 2,148 = 3,526



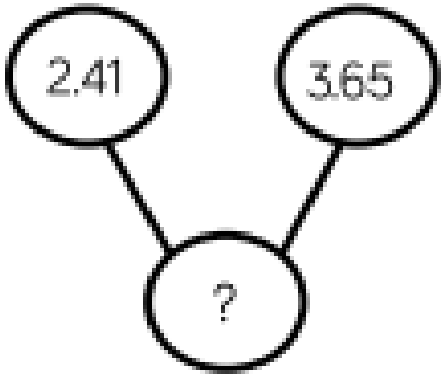


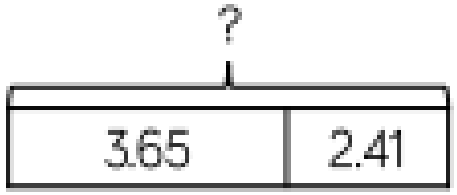
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

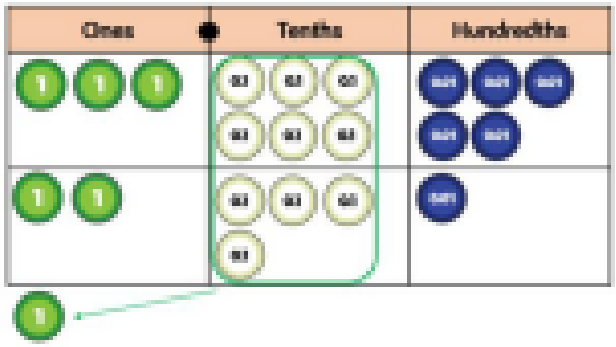
Skill: Add with up to 3 decimal places

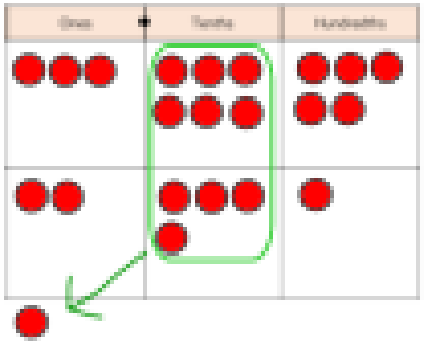




$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

3.65 + 2.41 = 6.06





Year: 5

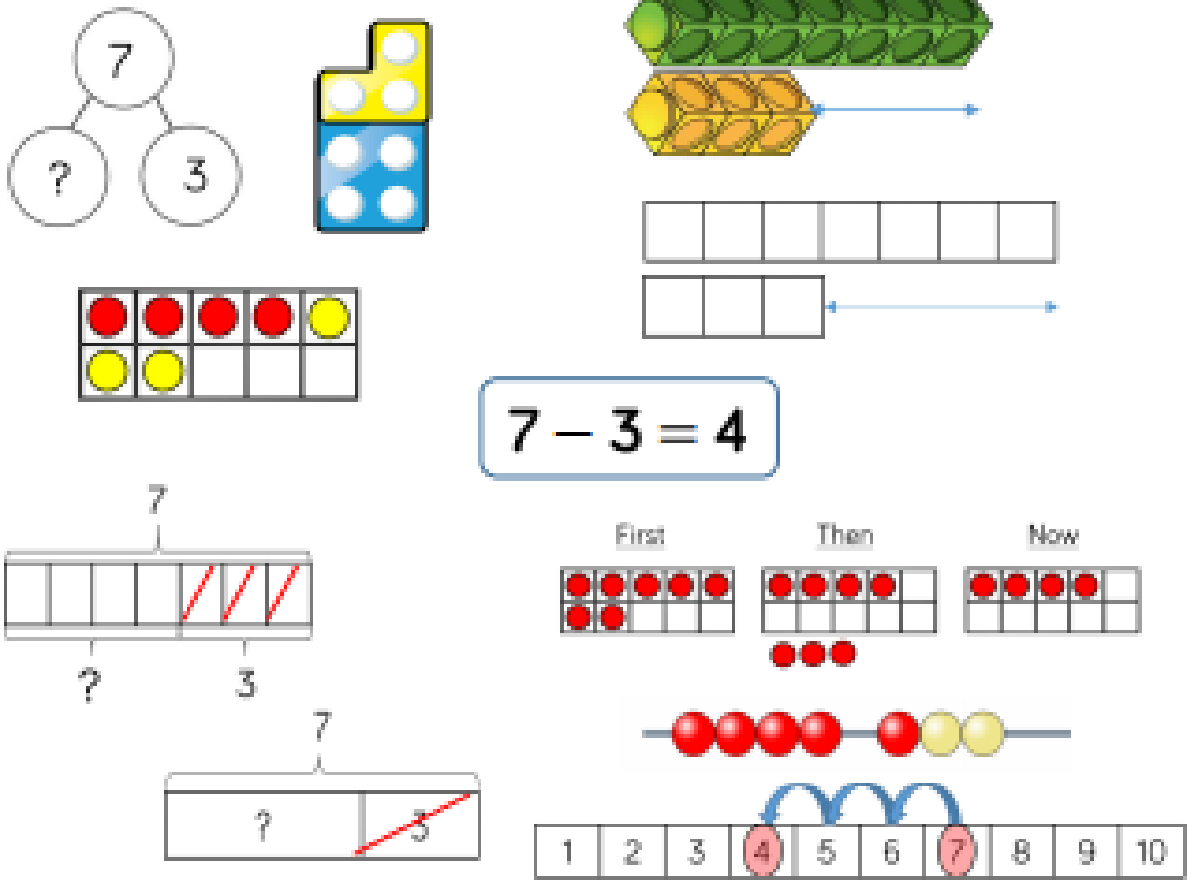
Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

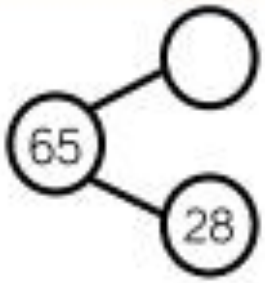
SUBTRACTION

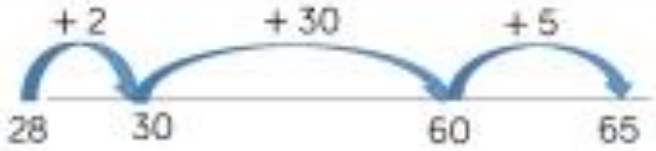
Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead string (20) Number tracks Number lines (labelled) Straws
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition
Subtract with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Subtract with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

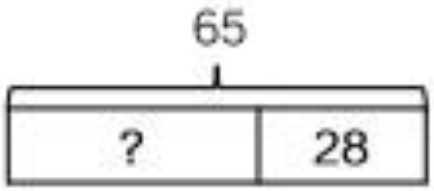
Subtraction

Skill: Subtract 1-digit numbers within 10	Year: 1
	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.

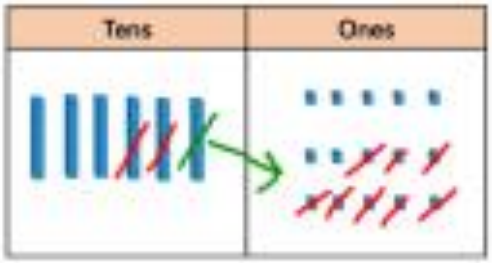
Skill: Subtract 1 and 2-digit numbers to 100



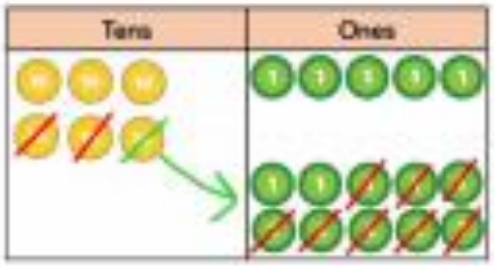




$65 - 28 = 37$



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}5 \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$$

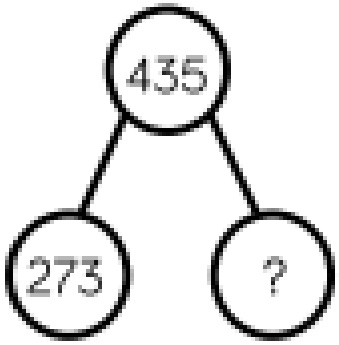


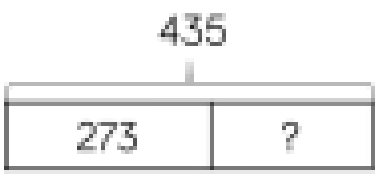
Year: 2

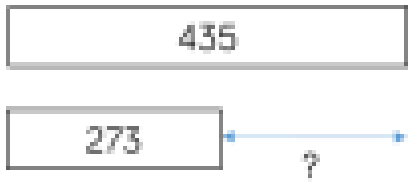
At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

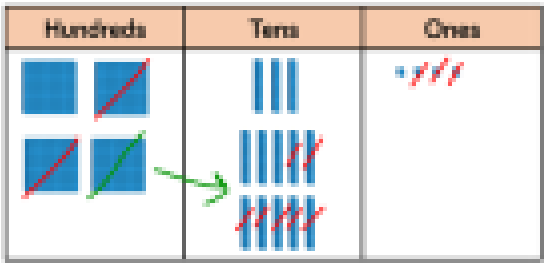
Skill: Subtract numbers with up to 3 digits	Year: 3



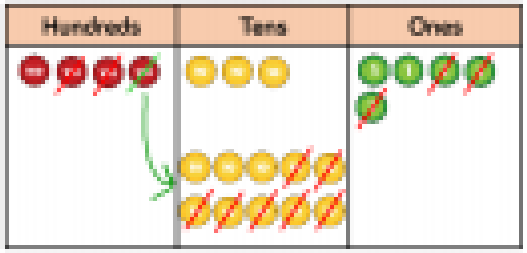




$$435 - 273 = 262$$



$$\begin{array}{r} {}^{3}\not{4}{}^{1}\not{3}5 \\ -\ 273 \\ \hline 262 \end{array}$$

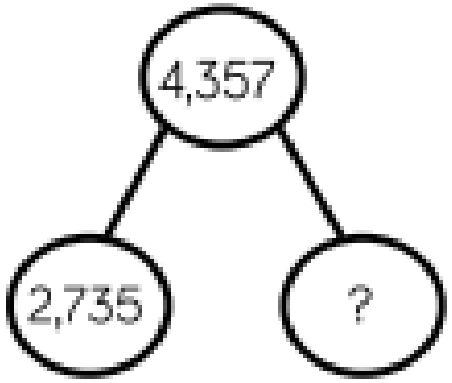


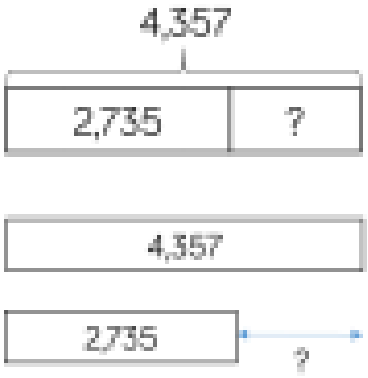
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

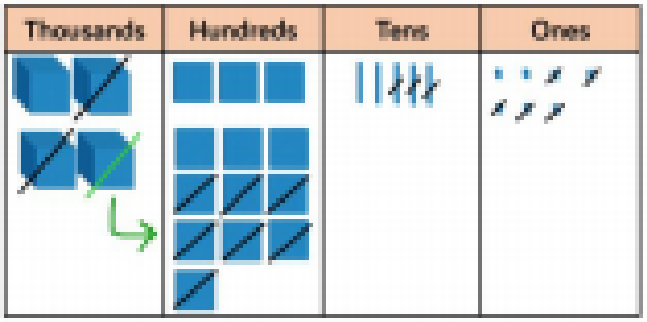
Skill: Subtract numbers with up to 4 digits

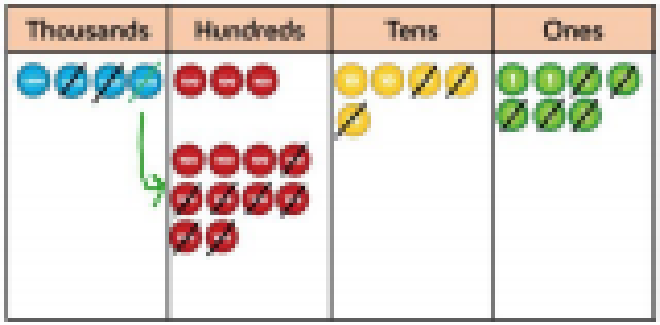




$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

4,357 − 2,735 = 1,622





Year: 4

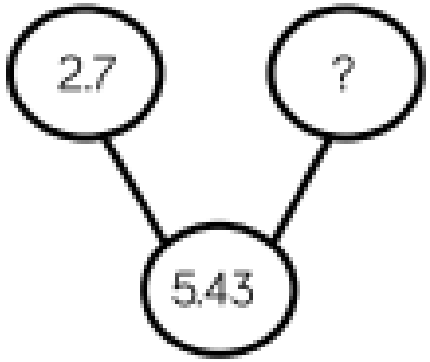
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

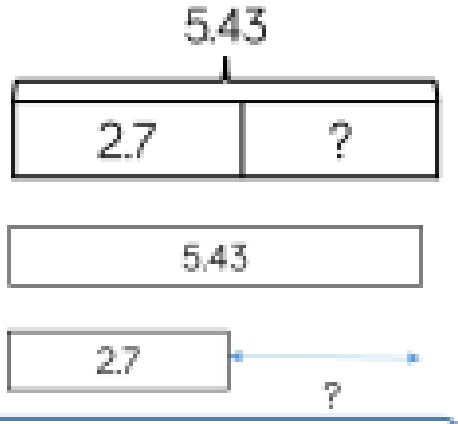
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Subtract with up to 3 decimal places

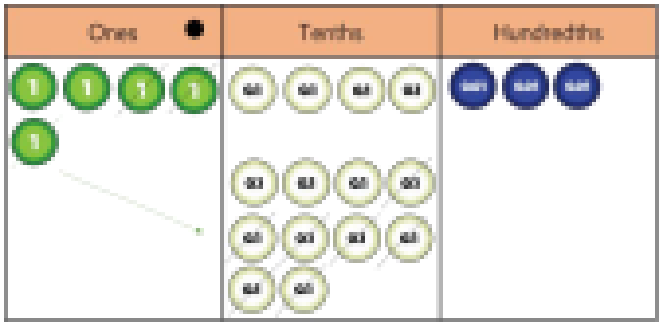
Year: 5

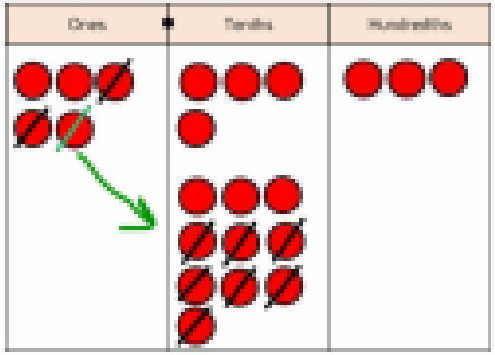




$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$





Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

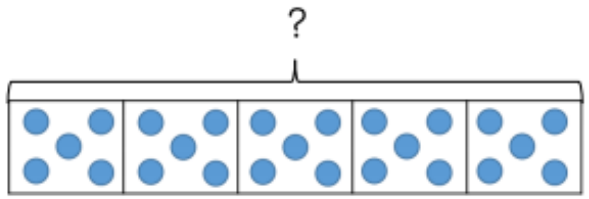
Total – The aggregate or the sum found by addition.

Multiplication and Division Strategies

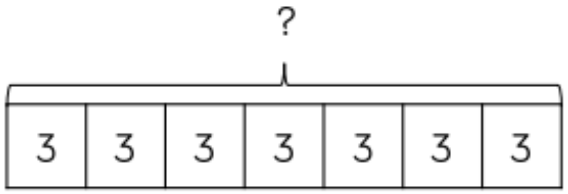
Modelling

The following methods of modelling are used when teaching addition and subtraction. There is an example of each technique taught and an explanation as to the benefits of each method.

Bar Model

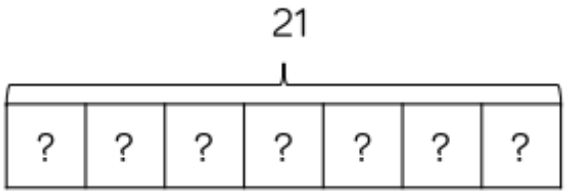


$5 \times 5 = 25$

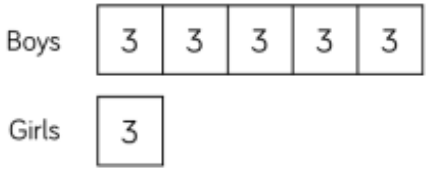


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits of this method:

This single bar model can be used to represent multiplication as repeated addition.

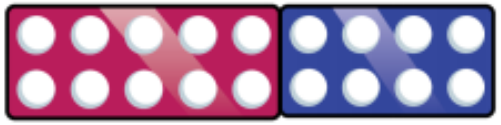
Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving a word problem that the bar model represents the problem.

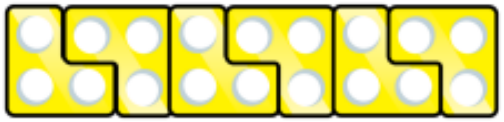
Number Shapes

$5 \times 4 = 20$
$4 \times 5 = 20$

$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Benefits of this method:

Number shapes can be used to represent multiplication as repeated addition.

Children can build multiplication in rows using the number shapes.

This method can support children in finding patterns e.g. odd x odd = even, odd x even = odd and even x even = even.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

Benefits of this method:

Bead strings to 100 can support children in their understanding of multiplication as repeat addition.

The colour of the beads supports children in seeing the groups.

Children can use the beads to count forwards and backwards, moving the beads as they count.

When dividing, children build the number they are making and then divide it into groups.

Number Tracks

$6 \times 3 = 18$
$3 \times 6 = 18$

$18 \div 3 = 6$

Benefits of this method:

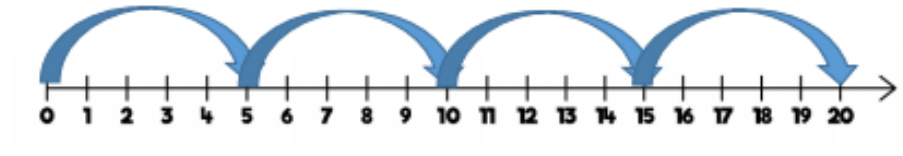
Number tracks are useful in helping children count in multiples, forwards and backwards. Translucent counters are the best so the children can see the number underneath.

When multiplying, the children start at 0 and count on to find the product of their number.

When dividing, the children place the counter on the number they are dividing and count back in jumps until they reach 0.

Number tracks are useful only for working with smaller numbers.

Number Lines (labelled)



$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

Benefits of this method:

Labelled number lines are useful for children counting in multiples, both forwards and backwards as well as calculating single digit multiples.

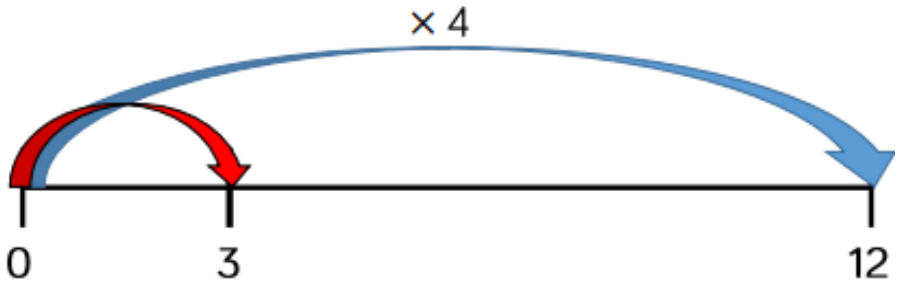
When multiplying, the children start at 0 and count on to find the product of their number.

When dividing, the children place the counter on the number they are dividing and count back in jumps until they reach 0.

Labelled number lines are useful only for working with smaller numbers.

Number Lines (blank)

A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



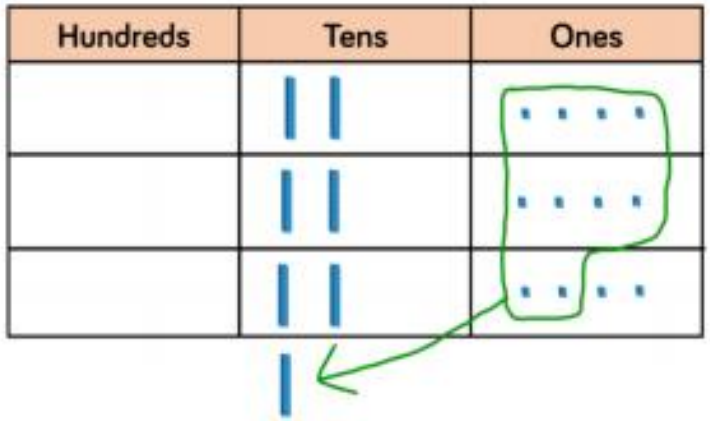
A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits of this method:

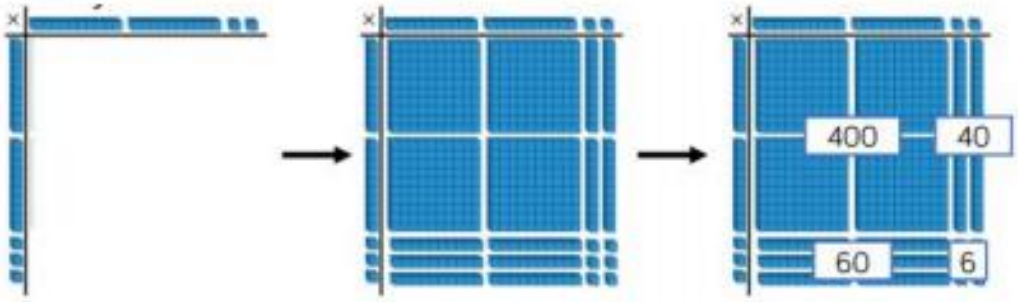
Children can use blank number lines to represent scaling as multiplication and division.

Blank number lines can be used to help represent scaling accurately and when there are no division children can practise accurate scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$

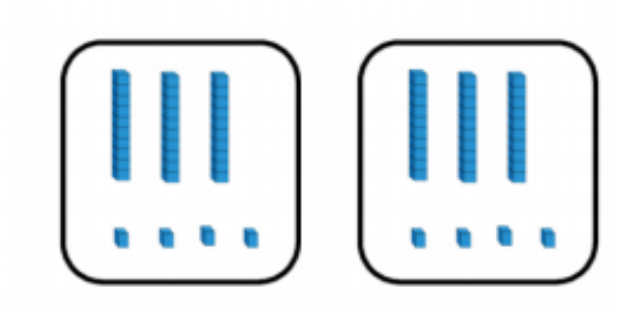


Benefits of this method:

Using base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation so they can see the link.

The use of Dienes is also good for representing the area model.

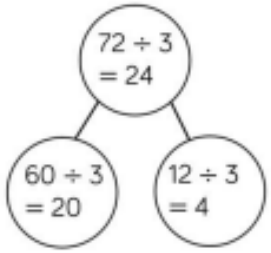
Base 10/Dienes (division)



$68 \div 2 = 34$

Tens	Ones

$72 \div 3 = 24$

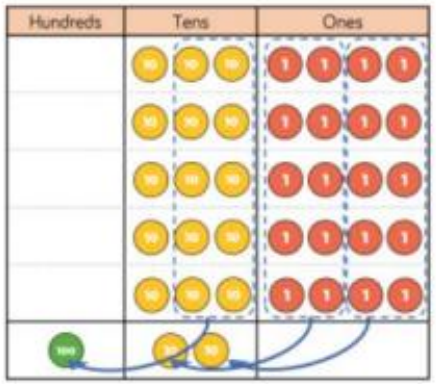


Benefits of this method:

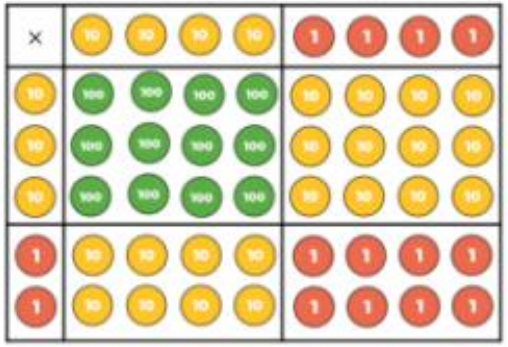
Using base 10 or Dienes is an effective way to support children's understanding of division. It is important that children write out their calculation so they can see the link.

Children should start with the left of the column for division.

Place Value Counters (multiplication)



$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline {\scriptstyle 1\ 2} \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline {\scriptstyle 1} \end{array}$$

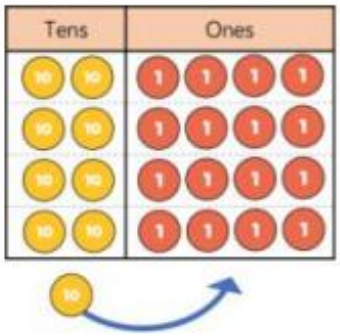
Benefits of this method:

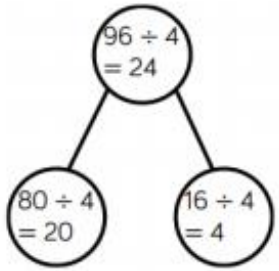
Using place value counters is an effective way to support children's understanding of column multiplication.

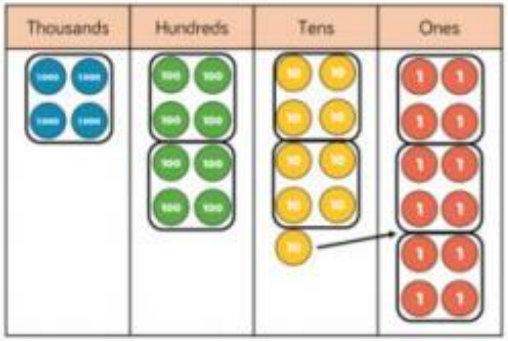
It is important that children write out their calculation so they can see the link.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division







$$\begin{array}{r} 1223 \\ 4\overline{)4892^{1}} \end{array}$$

Benefits of this method:

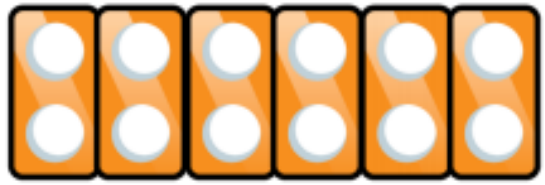
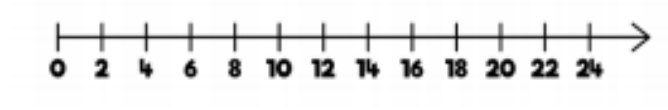
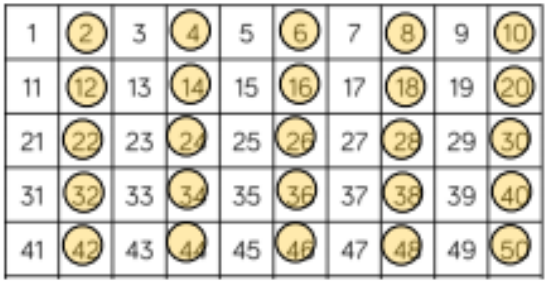
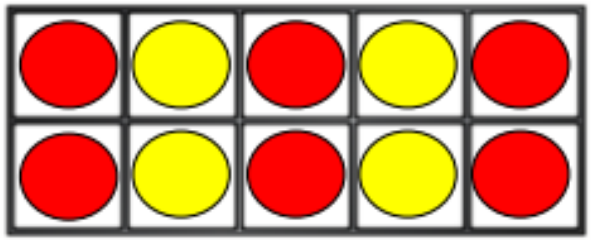
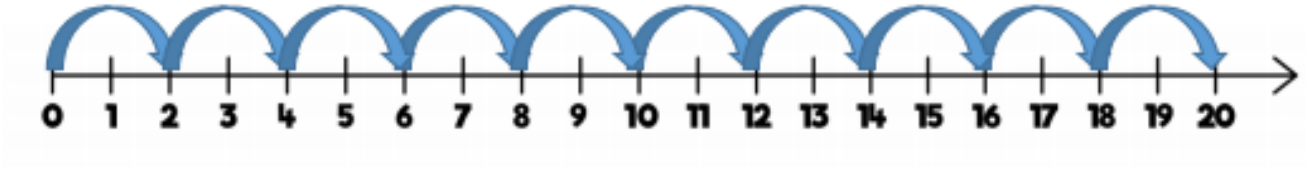
Using place value counters is an effective way to support children's understanding of division.

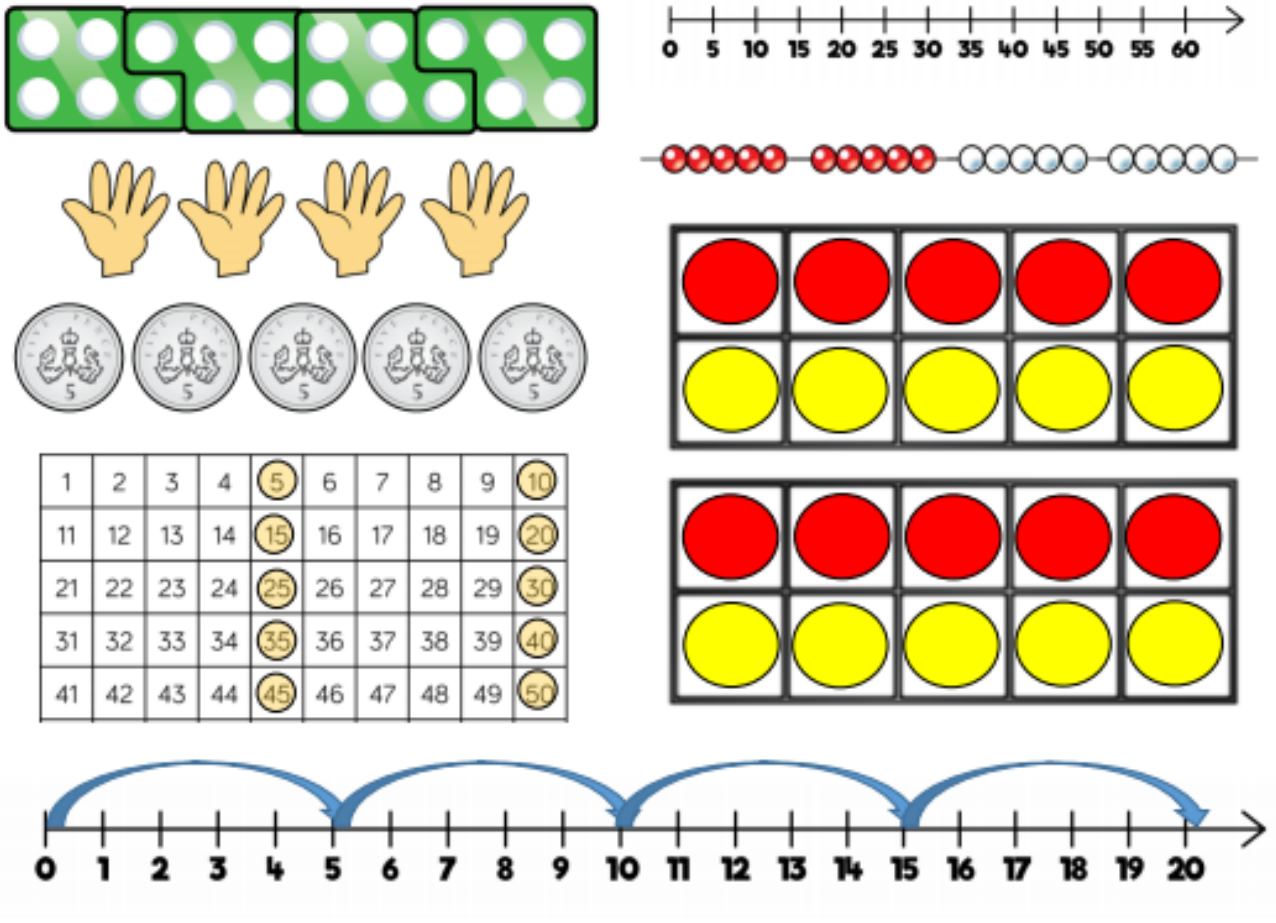
The children need to remember to work from the left to the right so exchange can occur if needed.

Place value counters give a better understanding of grouping rather than sharing.

Times Tables

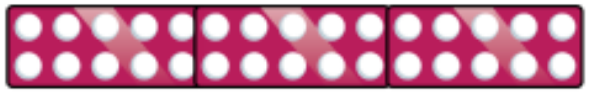
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 2-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 5-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 10-times table	2	Hundred square Number shapes Counters Money	Ten frames Bead strings Number lines Base 10
Recall and use multiplication and division facts for the 3-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 4-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 8-times table	3	Hundred square Number shapes	Bead strings Number tracks Everyday objects
Recall and use multiplication and division facts for the 6-times table	4	Hundred square Number shapes	Bead strings Number tracks Everyday objects
Recall and use multiplication and division facts for the 7-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 9-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 11-times table	4	Hundred square Base 10	Place value counters Number lines
Recall and use multiplication and division facts for the 12-times table	4	Hundred square Base 10	Place value counters Number lines

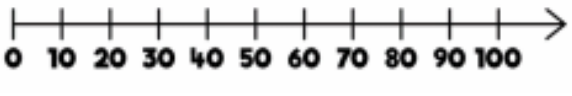
Skill: 2 times table	Year: 2
       	Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones. Use different models to develop fluency.

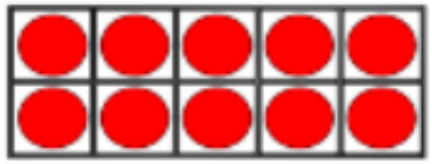
Skill: 5 times table	Year: 2
	Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

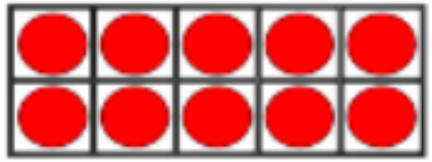
Skill: 10 times table

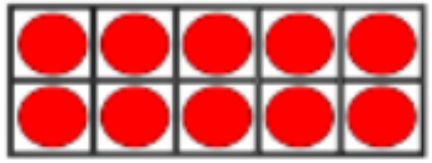
Year: 2







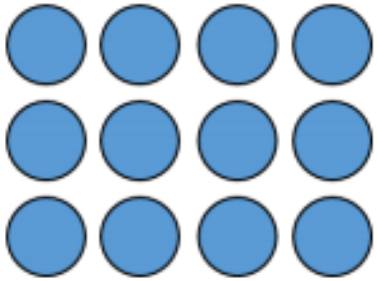
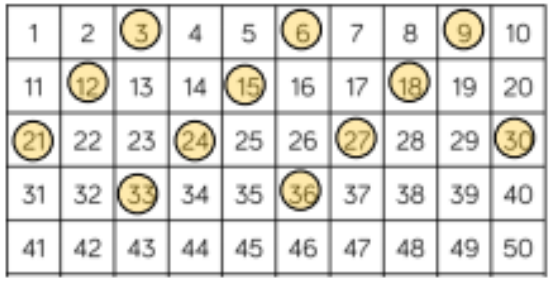
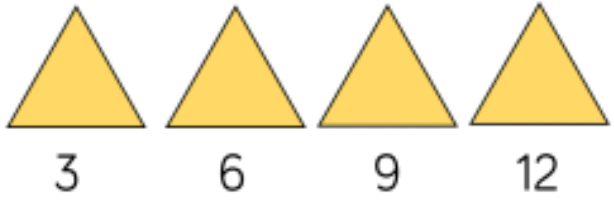
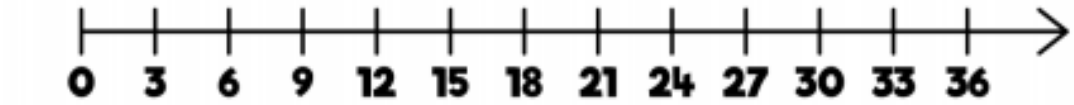


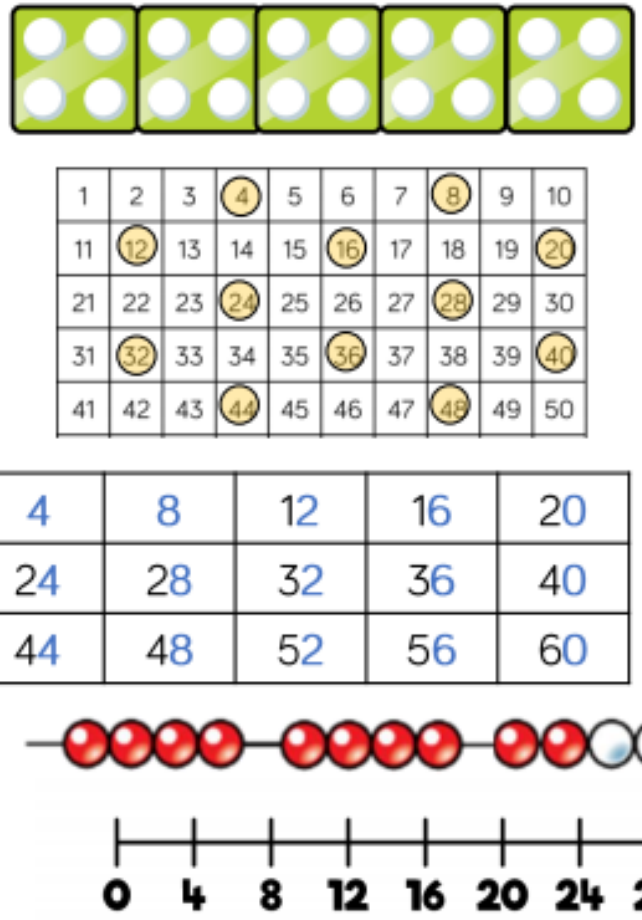


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

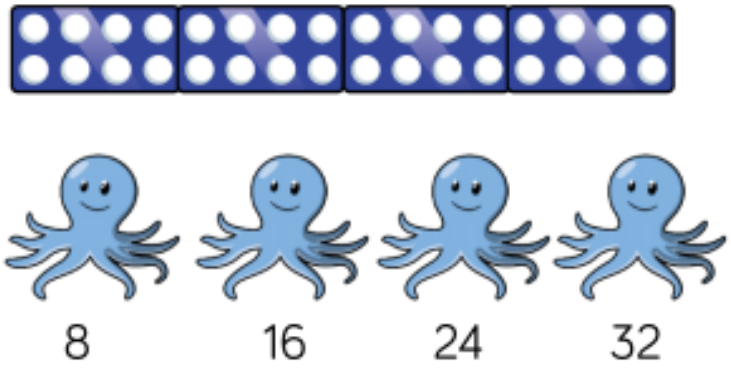
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits- the ones are always 0, and the tens increase by 1 ten each time.

Skill: 3 times table	Year: 3
     	Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

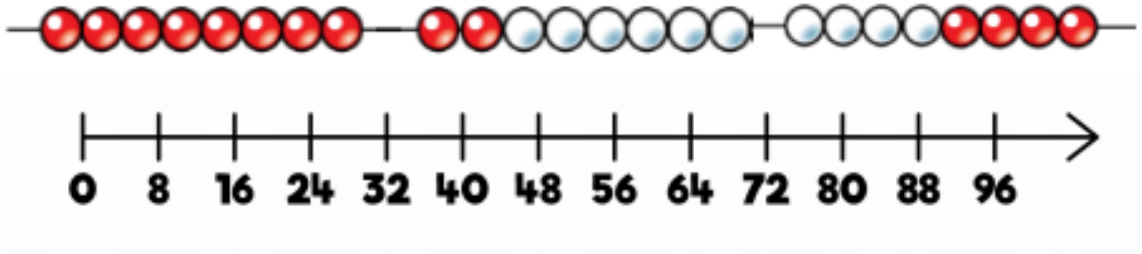
Skill: 4 times table	Year: 3
	Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 8 times table	Year: 3



8	16	24	32	40
48	56	64	72	80

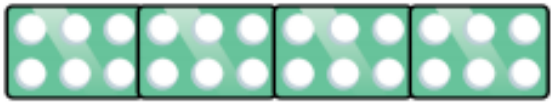
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

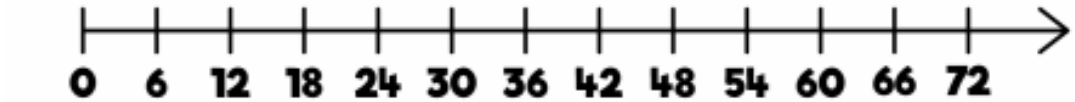
Skill: 6 times table

Year: 4



6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

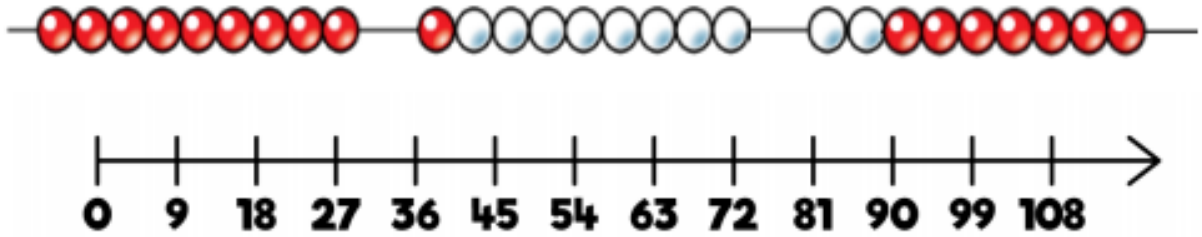


Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 9 times table	Year: 4

9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

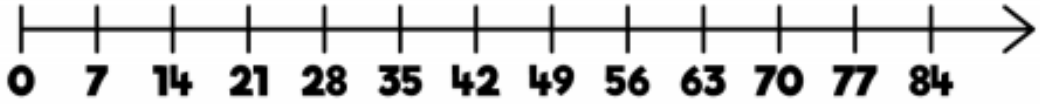


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

Skill: 7 times table	Year: 4

7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



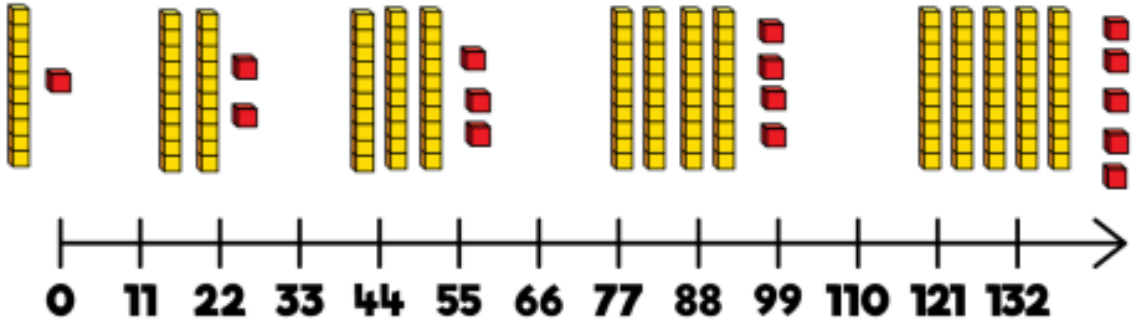
Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square.
The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity.
Children can still see the odd, even pattern in the multiples using number shapes to support.

Skill: 11 times table

Year: 4

11	22	33	44	55	66
77	88	99	110	121	132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

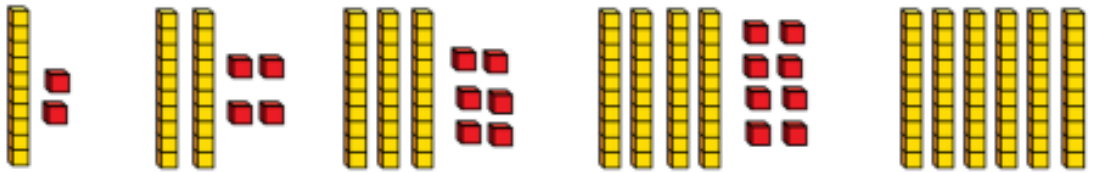
Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

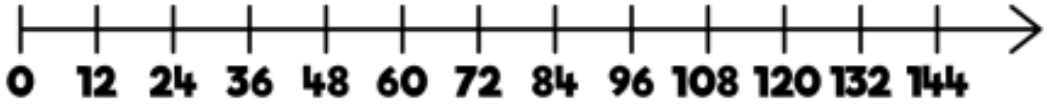
Skill: 12 times table

Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

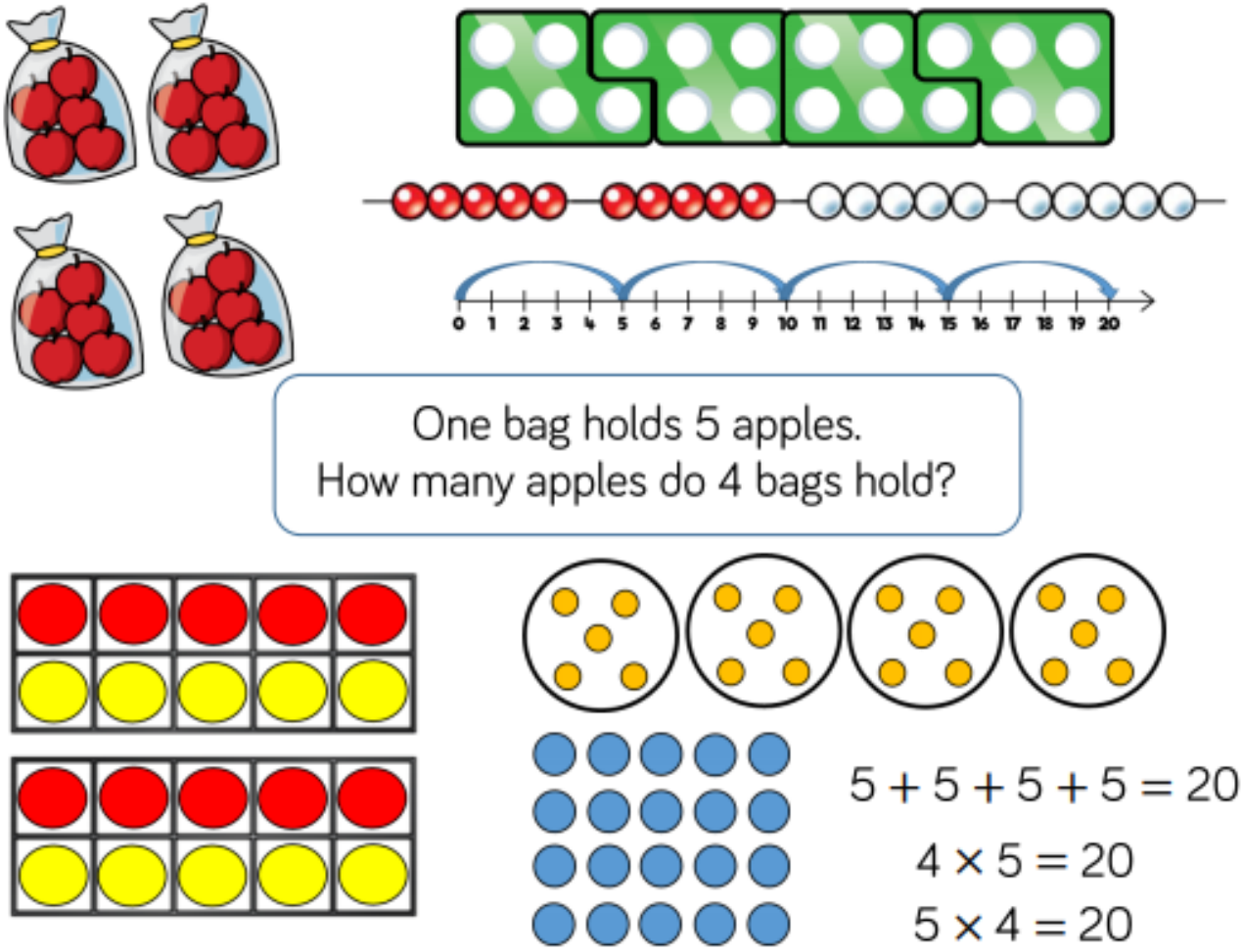




Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this

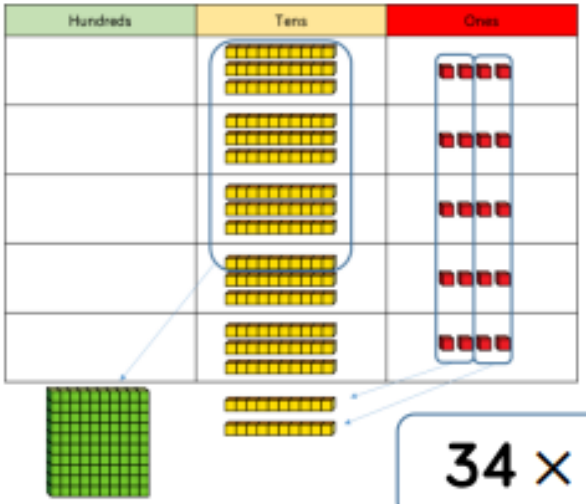
Multiplication

Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Short written method Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Skill: Solve 1-step problems using multiplication	Year: 1/2
	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

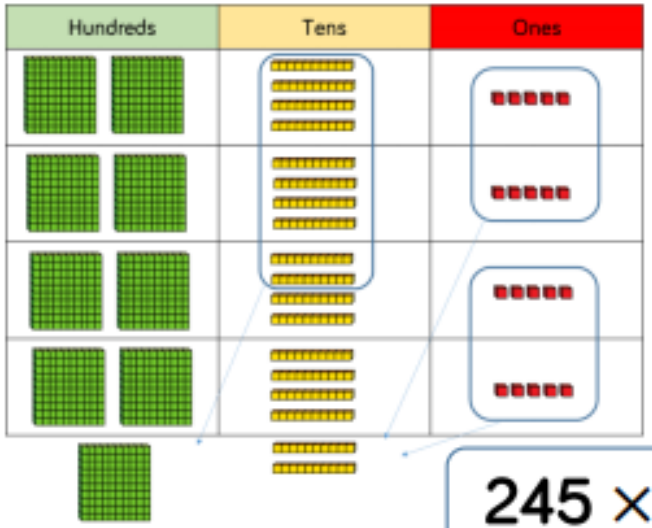
$$34 \times 5 = 170$$

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method.
The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

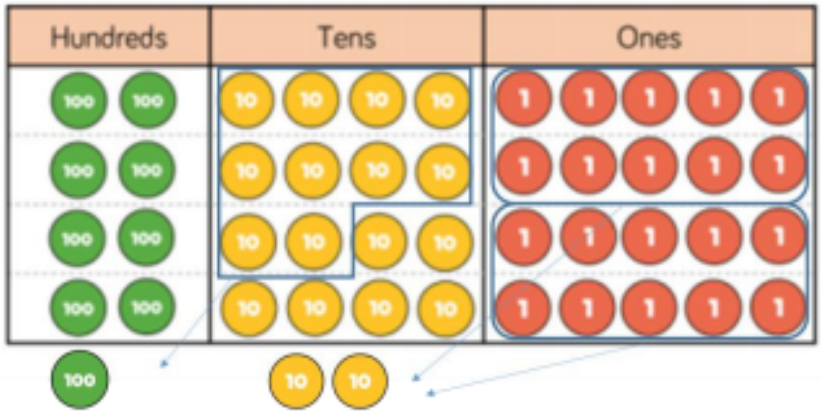
Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

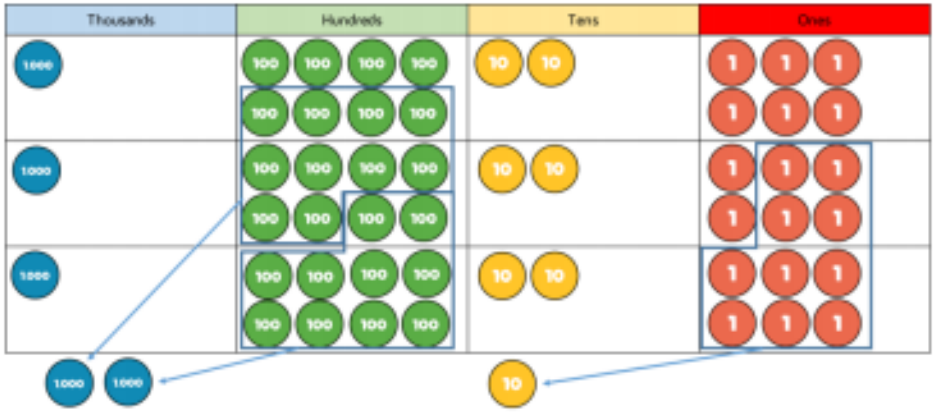
$$245 \times 4 = 980$$



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.
Base 10 and place value counters continue to support the understanding of the written method.
Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



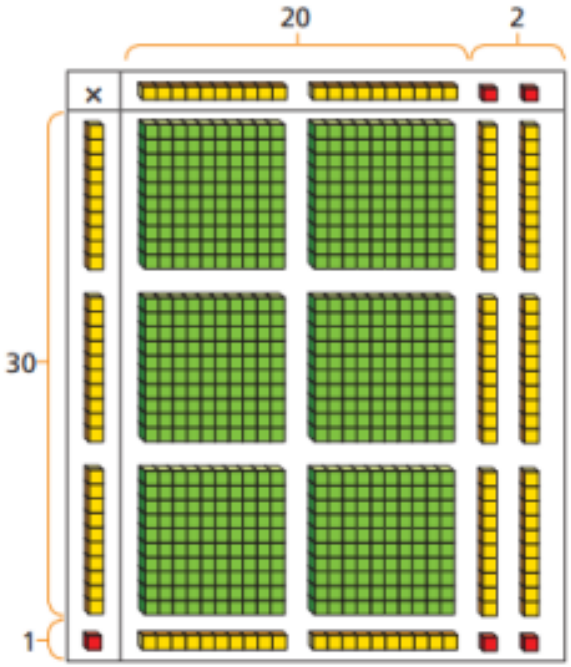
$$1{,}826 \times 3 = 5{,}478$$

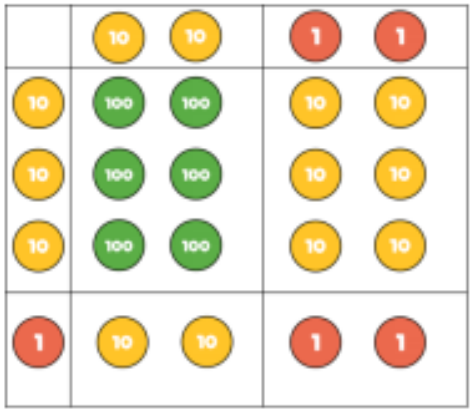
	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5





×	20	2
30	600	60
1	20	2

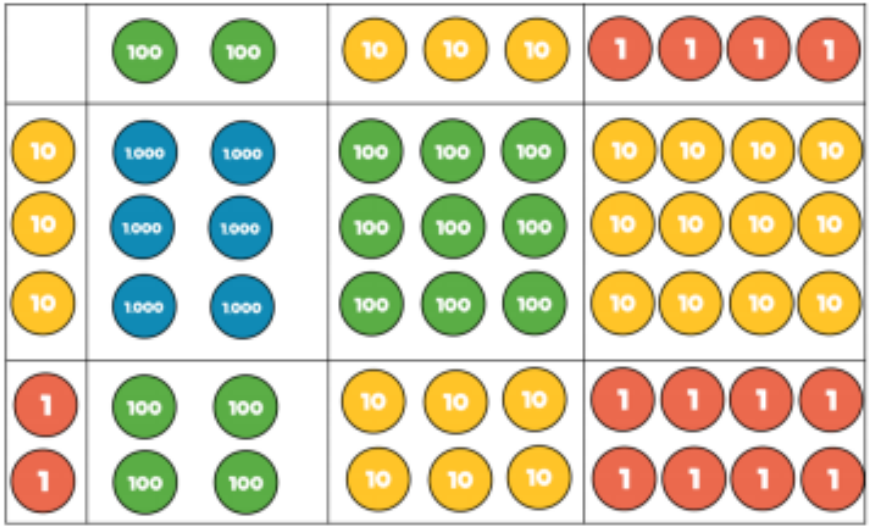
	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.
The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$^{1}7$	$^{1}0$	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$234 \times 32 = 7{,}488$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

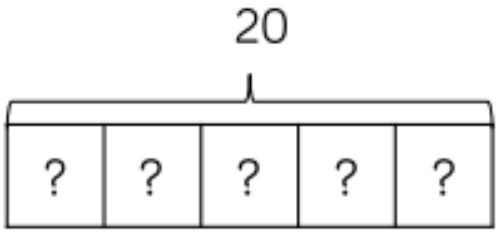
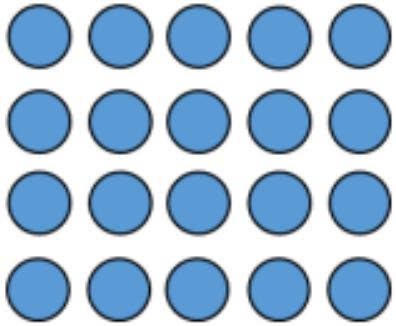
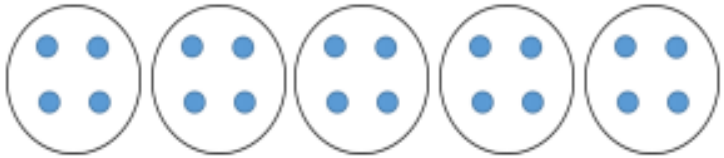
Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6
	When multiplying 4-digits by 2-digits, children should be confident in the written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.

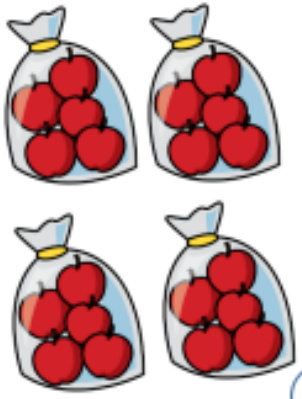
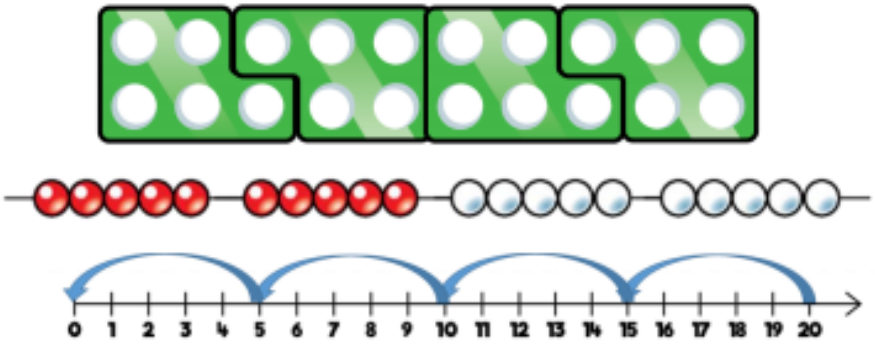
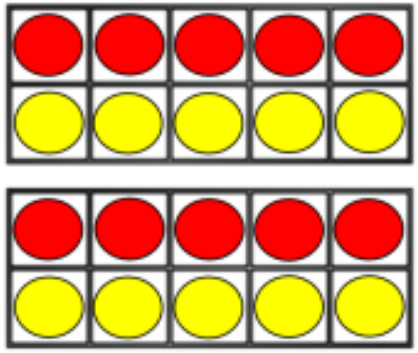
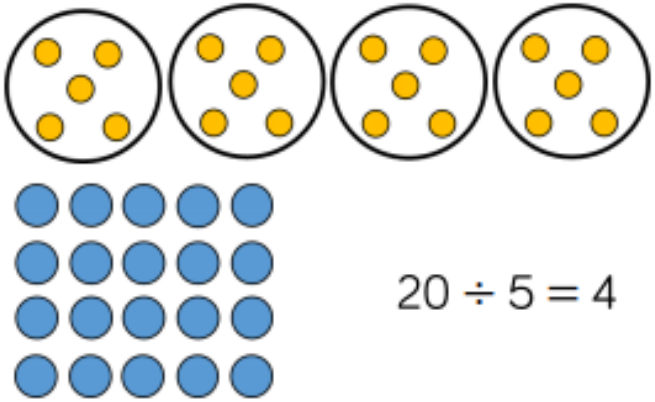
$2{,}739 \times 28 = 76{,}692$

Division

Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Skill	Year	Representations and models	
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

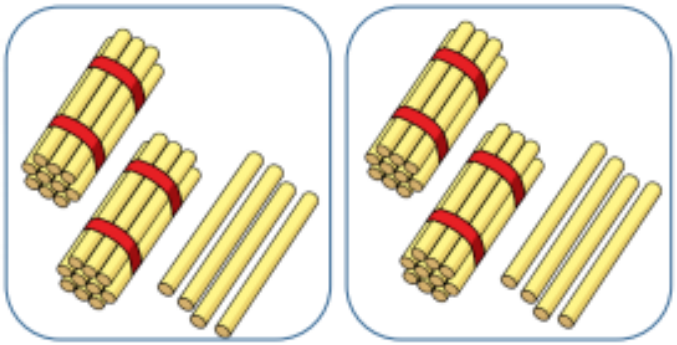
Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2
  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?   $20 \div 5 = 4$	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.

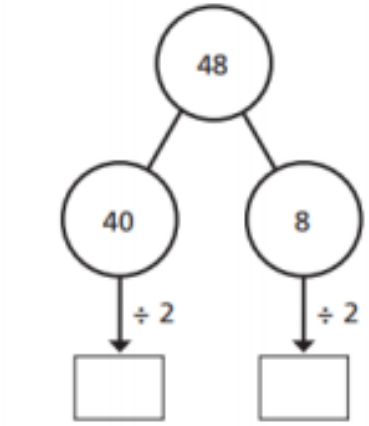
Skill: Solve 1-step problems using division (grouping)	Year: 1/2
  There are 20 apples altogether. They are put in bags of 5. How many bags are there?   $20 \div 5 = 4$	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

Skill: Divide 2-digits by 1-digit (sharing with no exchange)

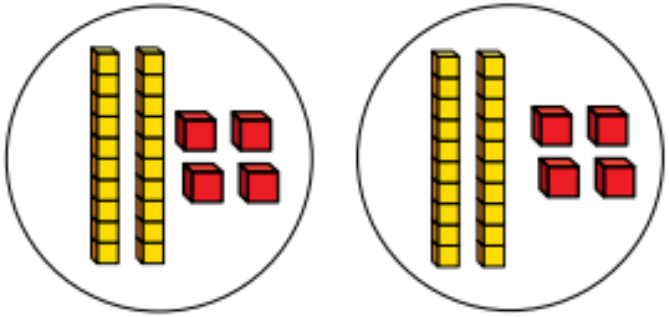
Year: 1/2

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1





$$48 \div 2 = 24$$



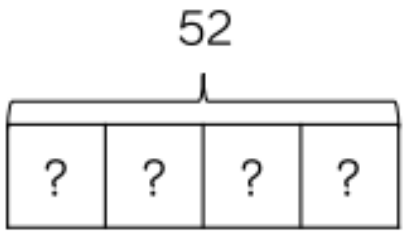
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

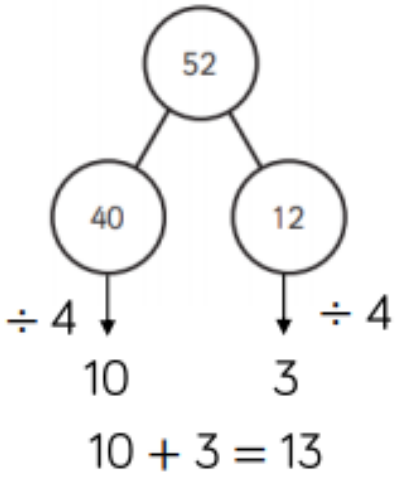
Part-whole models can provide children with a clear written method that matches the concrete representation.

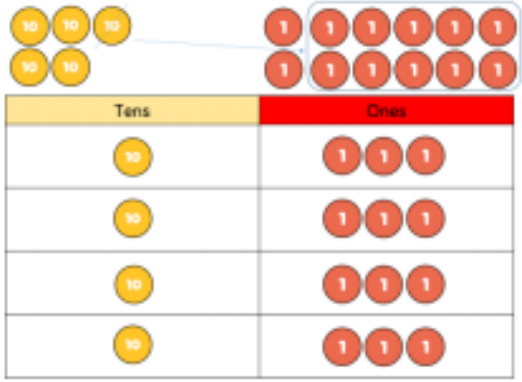
Skill: Divide 2-digits by 1-digit (sharing with exchange)

Year: 3/4



$$52 \div 4 = 13$$



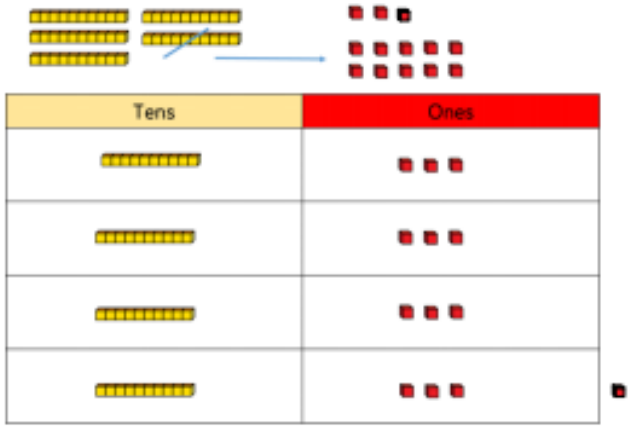


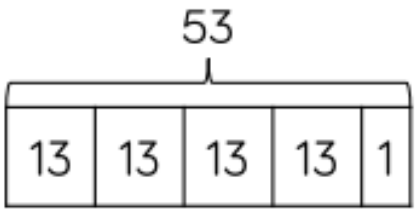
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

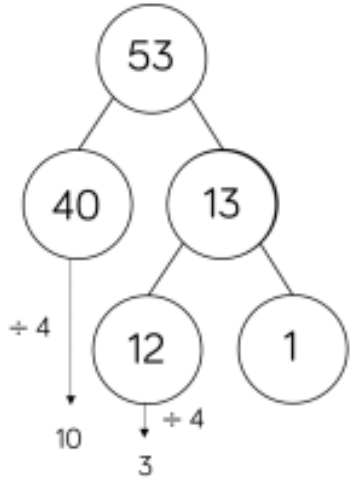
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4





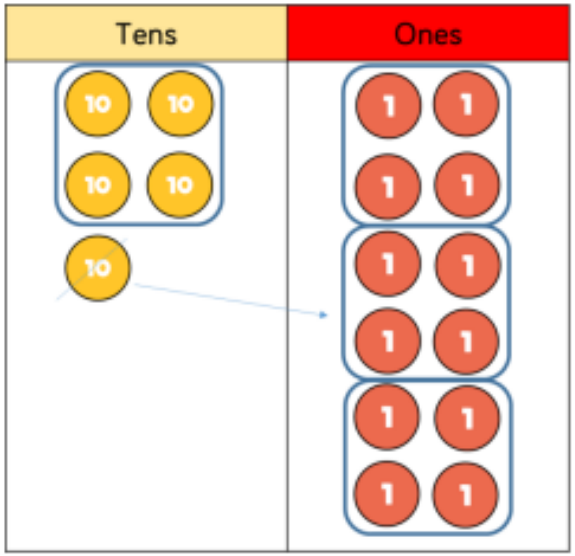


$$53 \div 4 = 13 \text{ r}1$$

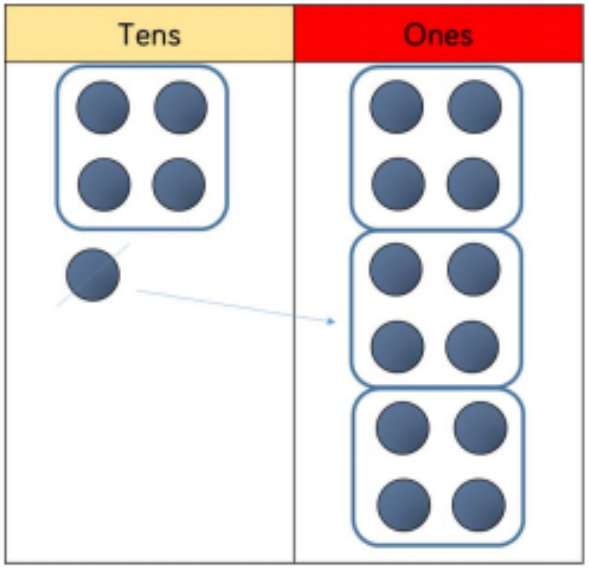
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (grouping)

Year: 4/5



		1	3	
	4	5	1 2	



$52 \div 4 = 13$

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

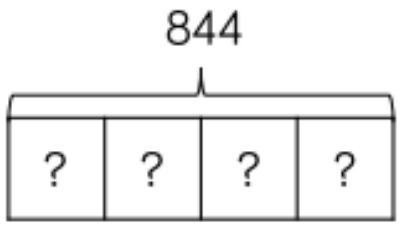
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

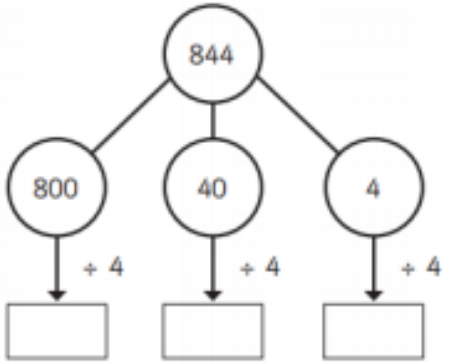
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

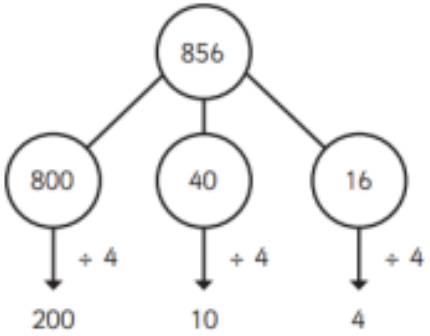
844 ÷ 4 = 122

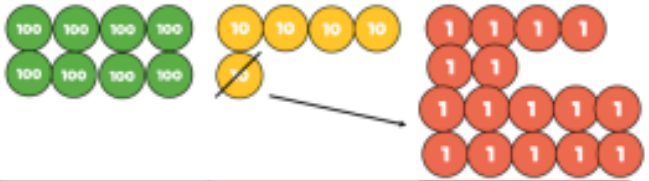


H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



844 ÷ 4 = 122



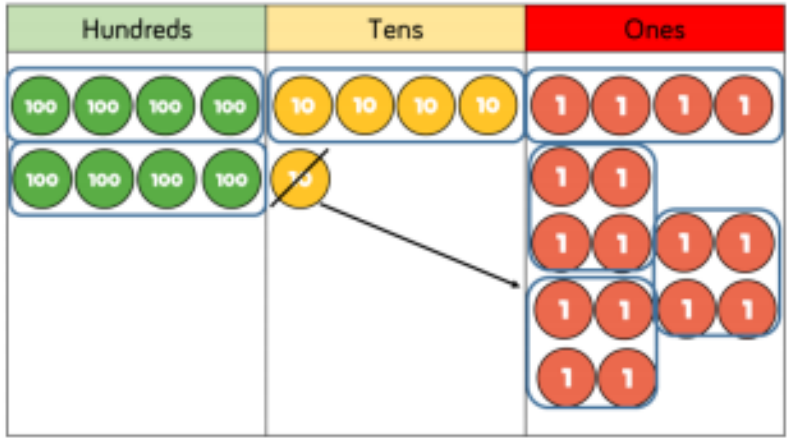


Hundreds	Tens	Ones
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1

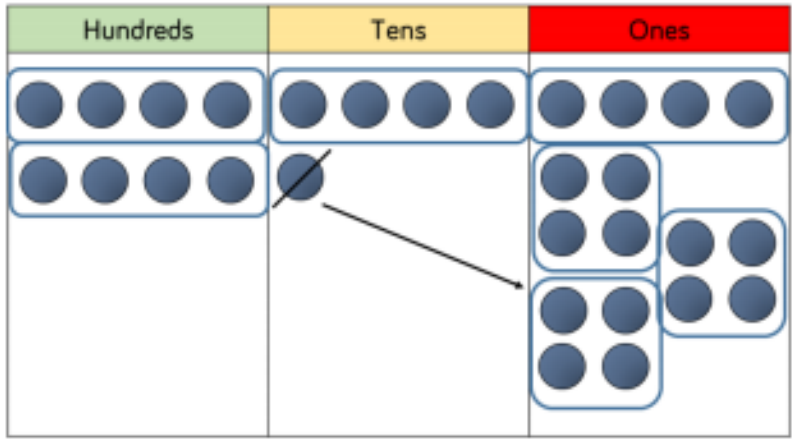
Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 3-digits by 1-digit (grouping)

Year: 5



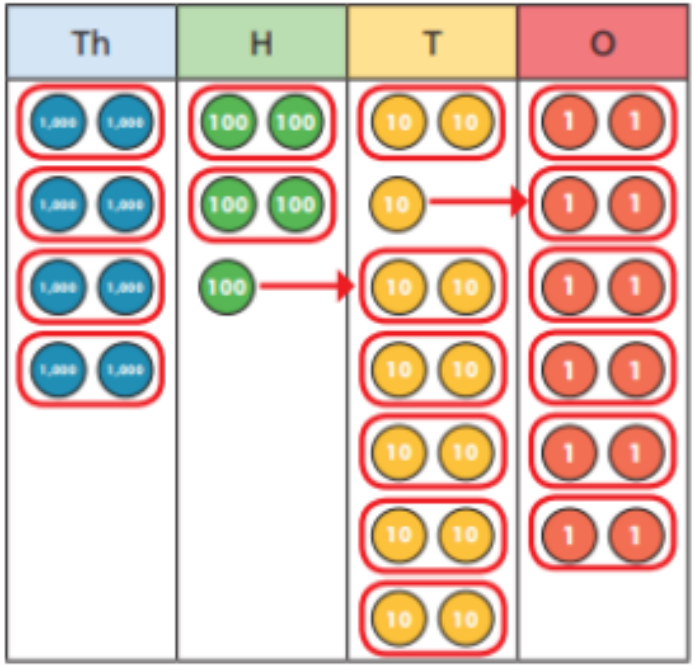
		2	1	4
	4	8	5	$^{1}6$



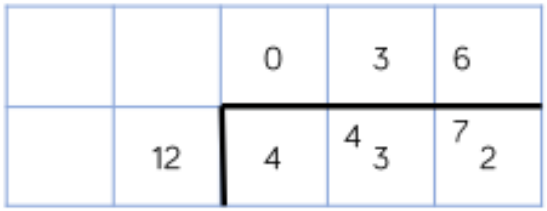
$856 \div 4 = 214$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)	Year: 5
 8,532 ÷ 2 = 4,266	Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

Skill: Divide multi digits by 2-digits (short division)	Year: 6
 $432 \div 12 = 36$  $7{,}335 \div 15 = 489$ 15 30 45 60 75 90 105 120 135 150	When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long division)

Year: 6

		0	3	6
1	2	4	3	2
	–	3	6	0
			7	2
	–		7	2
				0

(×30)

(×6)

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

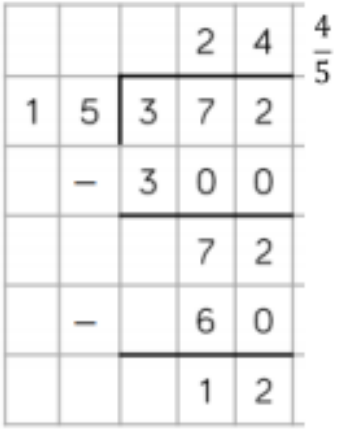
$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)	Year: 6
$372 \div 15 = 24\ r12$ $1 \times 15 = 15$ $2 \times 15 = 30$ $3 \times 15 = 45$ $4 \times 15 = 60$ $5 \times 15 = 75$ $10 \times 15 = 150$ $372 \div 15 = 24\frac{4}{5}$	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.



Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor